

**Estudio Analítico - Gráfico
de los
Poliedros Arquimedianos**

LAMINAS 68 AL 85

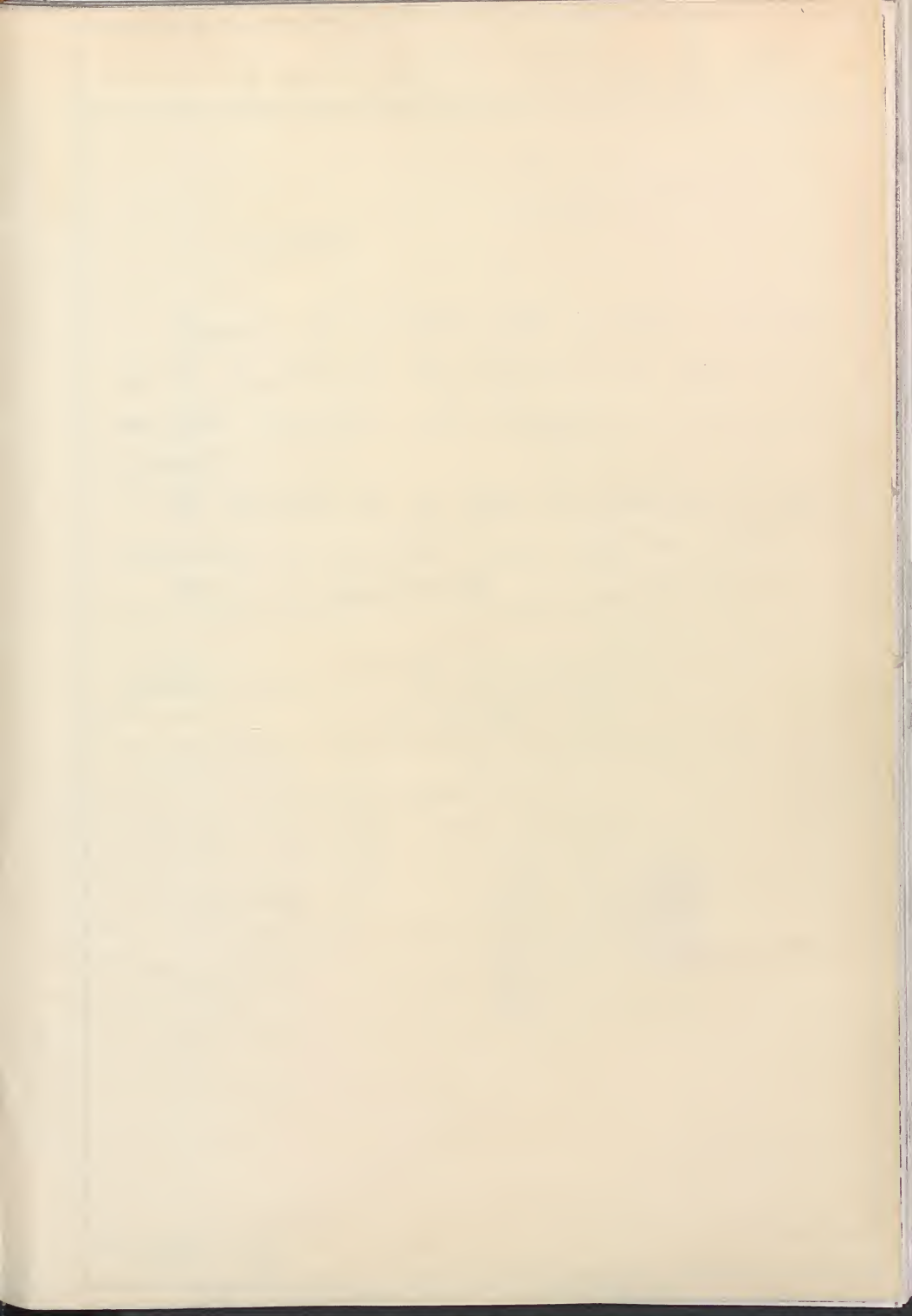
VI

VI

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Prof. T. Álvarez Peralto





R. 7815

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ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimedianos I, en el que en cada vértice concurren cuatro triángulos equiláteros y un cuadrado.

La longitud de su lado es de 40,9 mm, y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1.

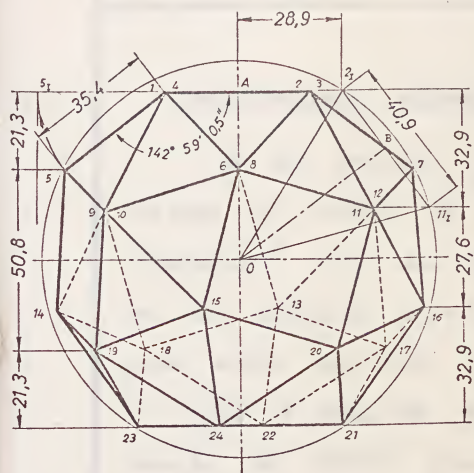
DATOS

O (72, 72, 85) mm

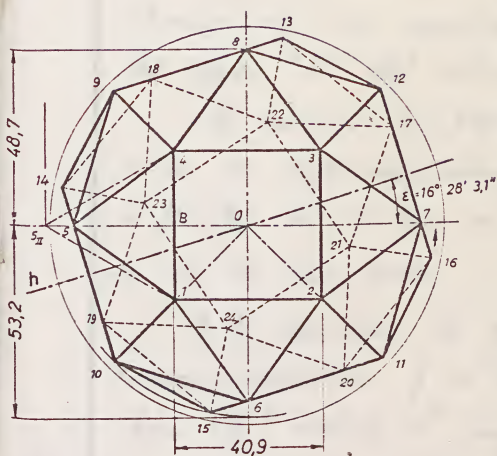
 $l_1 = 40,9$ mm.A
514
ALV
VI



I



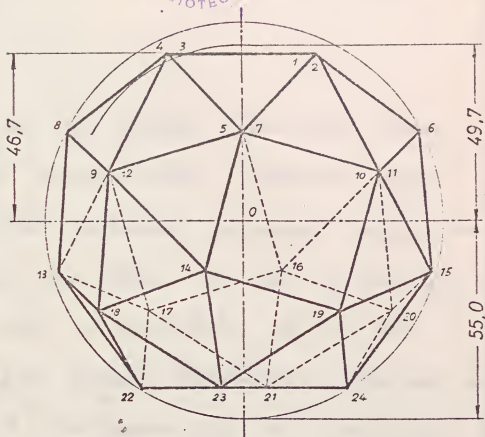
+X



+Y



III



+Y

ARQUIMEDIANO I

Número de caras triangulares..... $C_3 = 32$
 Número de caras cuadradas..... $C_4 = 6$
 Número de vértices..... $V = 24$
 Número de aristas..... $A = 60$
 Número de caras de un ángulo sólido: $4C_3 + 1C_4$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano I, en el que en cada vértice concurren cuatro triángulos equiláteros y un cuadrado.

La longitud de su lado es de 40,9 mm, y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

II

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Arquimediano I					Lámina 33
1:1						Curso 19 - 19



THESE TWO FIGURES
 REPRESENT THE
 SAME FIGURE
 IN TWO DIFFERENT
 POSITIONS
 THE FIRST FIGURE
 IS A CIRCLE
 WITH AN INSCRIBED
 POLYGON
 THE SECOND FIGURE
 IS A CIRCLE
 WITH AN INSCRIBED
 POLYGON
 THE TWO FIGURES
 ARE IDENTICAL
 IN EVERY RESPECT
 EXCEPT THEIR
 POSITION



CONSIDERACIONES PREVIAS

En esta lámina y en las catorce siguientes vamos a realizar el estudio de los denominados "Poliedros arquimedianos" que son aquellos poliedros convexos cuyas caras son polígonos regulares no todos iguales y sus ángulos sólidos son todos iguales o simétricos dos a dos.

En el desarrollo de este trabajo tendremos siempre en cuenta el estudio general realizado para estos poliedros, en el que se detalla los casos posibles de existencia, propiedades geométricas y fórmulas generales para la obtención de sus principales magnitudes, en función del lado "l" del poliedro que se toma como dato.

El número de poliedros arquimedianos posibles es el de 13 tipos individuales, a más de dos series infinitas en el que es variable el número "n" de lados de dos de sus caras.

A los primeros los hemos designado con cifras romanas sucesivas y a los segundos "Serie A_n" y "Serie B_n" siendo "n" un entero mayor de 3 en la primera y mayor de 2 en la segunda.

Las fórmulas generales deducidas en el mencionado estudio y que aplicaremos a cada caso particular, son las siguientes:

$$a = \frac{l^2}{2\sqrt{l^2 - m^2}} \quad [1]$$

$$c = \sqrt{a^2 - d^2} \quad [2]$$

$$b = \sqrt{a^2 - \frac{p^2}{4}} \quad [3]$$

$$\varphi_{pq} = \alpha_p + \beta_q \quad [4]$$

$$\tan \alpha_p = \frac{2c_p}{\sqrt{4(d_p)^2 - l^2}} \quad [5]$$

$$\tan \alpha_q = \frac{2c_q}{\sqrt{4(d_q)^2 - l^2}} \quad [6]$$

en las que:

l = Arista del poliedro arquimediano (dato del problema)

a = Radio de la esfera circunscrita

b = Radio de la esfera tangente a las aristas.

c_p = Radio de la esfera tangente a las caras regulares de "p" lados

c_q = Radio de la esfera tangente a las caras regulares de "q" lados.

φ_{pq} = Ángulo rectilíneo del diedro formado por la

cara regular de "p" lados con la de "q" lados.

d_p = Radio de la circunferencia ~~circunscrita~~ circunscrita a una cara de "p" lados.

d_q = Id. id. a una cara de "q" lados.

m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido

También obtendremos para cada caso particular

S = Superficie

V = Volumen.

Para la representación de cada arquimédiano, nos valdremos principalmente de los valores obtenidos analíticamente por lo cual reduciremos el orden de exposición seguido en las láminas anteriores, limitándonos exclusivamente al "Proceso gráfico-analítico"

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimédiano, nos indica que tiene 32 caras regulares triangulares y 6 caras cuadradas; 24 vértices y 60 aristas.

En cada vértice concurren 4 caras triangula-



1. The first part of the paper is devoted to a general
discussion of the problem. It is shown that the
problem is of great importance and that it has
not been completely solved. The author then
presents a new method for solving the problem.
The method is based on the use of the
Fourier transform and the method of steepest
descent. The author shows that the method
is very efficient and that it can be used to
solve a wide variety of problems.

2. In the second part of the paper, the author
applies the method to the problem of the
diffusion of a gas. It is shown that the
method can be used to calculate the rate of
diffusion of a gas through a porous medium.
The author also shows that the method can be
used to calculate the rate of diffusion of a
gas through a liquid. The results of the
calculations are compared with the results of
experiments and it is shown that the method
gives a very good agreement with the
experimental results.

3. In the third part of the paper, the author
applies the method to the problem of the
diffusion of a liquid. It is shown that the
method can be used to calculate the rate of
diffusion of a liquid through a porous medium.
The author also shows that the method can be
used to calculate the rate of diffusion of a
liquid through a liquid. The results of the
calculations are compared with the results of
experiments and it is shown that the method
gives a very good agreement with the
experimental results.

4. In the fourth part of the paper, the author
applies the method to the problem of the
diffusion of a solid. It is shown that the
method can be used to calculate the rate of
diffusion of a solid through a porous medium.
The author also shows that the method can be
used to calculate the rate of diffusion of a
solid through a solid. The results of the
calculations are compared with the results of
experiments and it is shown that the method
gives a very good agreement with the
experimental results.

es, 1 cara cuadrada y, por consiguiente, 5 aristas del mismo.
Así pues, tendremos que

$$\text{Arquimediano I } (4 P_3 + 1 P_4); C_3 = 32; C_4 = 6; V = 24; A = 60$$

Cálculo de sus magnitudes

Arista "l" del arquimediano

Dato del ejercicio

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las cinco aristas de un ángulo poliedro.

Este polígono plano será un pentágono irregular (concurren en el vértice 4 triángulos equiláteros y un cuadrado) formado por cuatro lados iguales y uno desigual.

Los cuatro lados iguales tienen una longitud igual a la arista "l" (tercer lado de cada cara triangular regular) y el quinto tendrá una longitud igual a la diagonal del cuadrado de lado "l", o sea " $\sqrt{2} l$ ".

En la figura 1 se representa este polígono, que

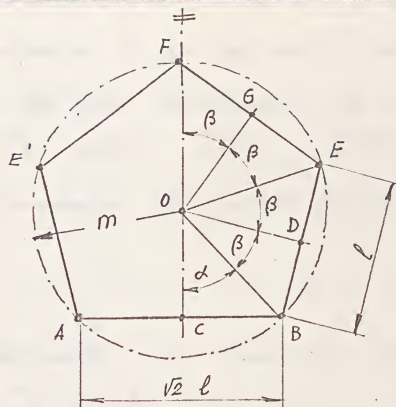


Figura 1

ha de estar inscrito en una circunferencia, cuyo radio "m" queremos determinar.

No existe solución gráfica exacta para este problema.

La solución analítica es la que estudiamos a continuación.

Suponiendo el problema resuelto* y refiriéndonos a la figura 1, sea O el centro de la circunferencia de radio "m" buscado (incógnita), en la cual está inscrito el pentágono A-B-E'-F-E' que tiene 4 lados iguales $BE = EF = FE' = E'A = l$ y el quinto desigual $AB = \sqrt{2}l$.

De la figura se deduce que el pentágono tiene un eje de simetría F-O-C, mediatriz del lado mayor AB.

Si unimos los vértices B y E con el centro O, se van a formar los triángulos isósceles B-O-E y E-O-F iguales, cuyas alturas con respecto a sus bases B-E y E-F, nos determinarán los puntos D y G respectivamente, que

* La existencia de la circunferencia circunscrita al polígono, que se presupone, la demostraremos posteriormente.

The first of these is the
 fact that the
 number of
 cases of
 the disease
 has been
 increasing
 steadily
 since the
 year 1880.



The second of these is the fact that the
 number of cases of the disease has been
 increasing steadily since the year 1880.
 The third of these is the fact that the
 number of cases of the disease has been
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 The seventh of these is the fact that the
 number of cases of the disease has been
 increasing steadily since the year 1880.
 The eighth of these is the fact that the
 number of cases of the disease has been
 increasing steadily since the year 1880.
 The ninth of these is the fact that the
 number of cases of the disease has been
 increasing steadily since the year 1880.
 The tenth of these is the fact that the
 number of cases of the disease has been
 increasing steadily since the year 1880.

uniremos a su vez con el centro O.

Así pues obtendremos el ángulo $\widehat{C \cdot O \cdot B} = \alpha$ y los $\widehat{B \cdot O \cdot D}$, $\widehat{D \cdot O \cdot E}$, $\widehat{E \cdot O \cdot F}$ y $\widehat{G \cdot O \cdot F}$ todos iguales, que denominaremos β , verificándose que

$$\alpha + 4\beta = 2\pi \quad (1)$$

El radio "m" buscado se puede obtener en función de " β " y "l", ya que

$$OB = \frac{BD}{\sin \beta} \quad \text{o sea:} \quad m = \frac{l:2}{\sin \beta} = \frac{l}{2 \sin \beta} \quad (2)$$

Para la determinación de " β " seguiremos el siguiente proceso de acuerdo con la fig. 1:

$$\sin \alpha = \frac{CB}{OB} = \frac{\frac{\sqrt{2} l}{2}}{m} = \frac{\sqrt{2} l}{2m} \quad (3)$$

por otra parte

$$\sin \beta = \frac{BD}{OB} = \frac{\frac{l}{2}}{m} = \frac{l}{2m} \quad (4)$$

y dividiendo (3) por (4)

$$\frac{\sin \alpha}{\sin \beta} = \frac{\frac{\sqrt{2} l}{2}}{\frac{l}{2}} = \sqrt{2} \quad (5)$$

Pero siendo el ángulo " α " suplementario del " 4β " se verificará que

Blank body area with faint horizontal lines.

$$\operatorname{sen} \alpha = \operatorname{sen} 4\beta \quad (6)$$

teniendo en cuenta (5) y (6), escribiremos

$$\boxed{\sqrt{2}} = \frac{\operatorname{sen} \alpha}{\operatorname{sen} \beta} = \frac{\operatorname{sen} 4\beta}{\operatorname{sen} \beta} = \boxed{8 \cos^3 \beta - 4 \cos \beta} \quad (7)$$

ecuación cúbica en " $\cos \beta$ " cuya solución nos determinara el valor de " $\cos \beta$ ", con el cual obtendríamos " $\operatorname{sen} \beta$ " y finalmente " m " por la ecuación (2)

Desarrollo del cálculo anterior:

$$\begin{aligned} \boxed{\sqrt{2}} &= \frac{\operatorname{sen} 4\beta}{\operatorname{sen} \beta} = \frac{\operatorname{sen} [2 \times (2\beta)]}{\operatorname{sen} \beta} = \frac{2 \operatorname{sen} 2\beta \cos 2\beta}{\operatorname{sen} \beta} = \\ &= \frac{2 \times 2 \operatorname{sen} \beta \cos \beta \times (2 \cos^2 \beta - 1)}{\operatorname{sen} \beta} = \frac{8 \operatorname{sen} \beta \cos^3 \beta - 4 \operatorname{sen} \beta \cos \beta}{\operatorname{sen} \beta} = \\ &= \boxed{8 \cos^3 \beta - 4 \cos \beta} \end{aligned}$$

Para resolver la ecuación cúbica (7), hagamos previamente $\cos \beta = x$, y escribiremos:

$$8x^3 - 4x = \sqrt{2}, \quad \text{de donde } 8x^3 - 4x - \sqrt{2} = 0$$

y dividiendo por 8, tendremos

$$x^3 - \frac{1}{2}x - \frac{\sqrt{2}}{8} = 0 \quad (8)$$

que se puede transformar en la general

$$z^3 + pz + q = 0$$

haciendo $z = x$, $p = -\frac{1}{2}$, $q = -\frac{\sqrt{2}}{8}$

La fórmula de Cardano* nos permite obtener "z" real, siempre que se verifique que

$$R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 > 0$$

lo cual sucede en este caso, ya que

$$R = \left(\frac{-\frac{\sqrt{2}}{8}}{2}\right)^2 + \left(\frac{-\frac{1}{2}}{3}\right)^3 = \frac{1}{128} - \frac{1}{216} = \frac{11}{3456} > 0$$

Desarrollo del cálculo anterior:

$$R = \left(\frac{-\frac{\sqrt{2}}{8}}{2}\right)^2 + \left(\frac{-\frac{1}{2}}{3}\right)^3 = \left(-\frac{\sqrt{2}}{16}\right)^2 + \left(-\frac{1}{6}\right)^3 = \frac{2}{16^2} - \frac{1}{3^3 \times 3^3} = \frac{1}{128} - \frac{1}{216} =$$

$$= \frac{2^3 \times 3^3 - 2^7}{2^{10} \times 3^3} = \frac{3^3 - 2^4}{2^7 \times 3^3} = \frac{11}{3456} > 0$$

y por consiguiente:

$$z = \sqrt[3]{-\frac{q}{2} + \sqrt{R}} + \sqrt[3]{-\frac{q}{2} - \sqrt{R}} = \sqrt[3]{-\left(-\frac{\sqrt{2}}{8}\right) \cdot 2 + \sqrt{\frac{11}{3456}}} + \sqrt[3]{-\left(-\frac{\sqrt{2}}{8}\right) \cdot 2 - \sqrt{\frac{11}{3456}}} =$$

$$= \sqrt[3]{\frac{\sqrt{2}}{16}} + \sqrt[3]{\frac{11}{3456}} + \sqrt[3]{\frac{\sqrt{2}}{16}} - \sqrt[3]{\frac{11}{3456}} = \boxed{0,84\ 25\ 09\ 20\ \dots} = \cos \beta \quad (9)$$

Desarrollo del cálculo anterior:

* Ver "Matemáticas para Ingenieros y Técnicos", de R. Doerfling, pag. 58.- Editorial Gustavo Gili, S.A., 1945.

No.	Date	Page
1	1914	1
2	1915	2
3	1916	3
4	1917	4
5	1918	5
6	1919	6
7	1920	7
8	1921	8
9	1922	9
10	1923	10
11	1924	11
12	1925	12
13	1926	13
14	1927	14
15	1928	15

$$\begin{aligned} \varphi &= \sqrt[3]{-\left(-\frac{\sqrt{2}}{8}\right):2} + \sqrt{\frac{11}{3456}} + \sqrt[3]{-\left(-\frac{\sqrt{2}}{8}\right):2} - \sqrt{\frac{11}{3456}} = \\ &= \sqrt[3]{\frac{\sqrt{2}}{16}} + \sqrt{\frac{11}{3456}} + \sqrt[3]{\frac{\sqrt{2}}{16}} - \sqrt{\frac{11}{3456}} \end{aligned}$$

$$\lg 2 = \underline{0,301\ 0300}$$

$$\lg \frac{2}{2} = 0,150\ 5150$$

$$- \lg 16 = \underline{-1,204\ 1200}$$

$$\lg \frac{\sqrt{2}}{16} = \underline{\underline{2,946\ 3950}}$$

$$\boxed{\frac{\sqrt{2}}{16} = 0,08\ 83\ 88\ 35}$$

$$\lg 11 = 1,041\ 3927$$

$$- \lg 3456 = \underline{-3,538\ 5737}$$

$$\underline{\underline{3,502\ 8190 =}}$$

$$= \underline{\underline{1,502\ 8190 - 4}}$$

$$\lg \sqrt{\frac{11}{3456}} = \frac{1,502\ 8190 - 4}{2} = 0,751\ 4095 - 2 = \underline{\underline{2,751\ 4095}}$$

$$\boxed{\sqrt{\frac{11}{3456}} = 0,05\ 64\ 16\ 94}$$

$$\frac{\sqrt{2}}{16} + \sqrt{\frac{11}{3456}} = 0,08\ 83\ 88\ 35 + 0,05\ 64\ 16\ 94 = 0,14\ 48\ 05\ 29$$

$$\frac{\sqrt{2}}{16} - \sqrt{\frac{11}{3456}} = 0,08\ 83\ 88\ 35 - 0,05\ 64\ 16\ 94 = 0,03\ 19\ 71\ 41$$

$$\lg 0,14\ 48\ 05\ 29 = \underline{\underline{7,160\ 7845 = 2,160\ 7845 - 3}}$$

$$\lg 0,03\ 19\ 71\ 41 = \underline{\underline{2,504\ 7618 = 1,504\ 7618 - 3}}$$

No.	Name	Age
1	John Smith	25
2	James Brown	30
3	William Jones	28
4	Robert Taylor	35
5	Thomas White	22
6	Charles Black	32
7	David Green	27
8	Richard Hill	38
9	Thomas Young	24
10	John King	31
11	James Lee	29
12	William Clark	33
13	Robert Adams	26
14	Thomas Baker	34
15	Charles Evans	23
16	David Wilson	36
17	Richard Scott	21
18	Thomas Green	37
19	John King	25
20	James Lee	30
21	William Clark	28
22	Robert Adams	35
23	Thomas Baker	22
24	Charles Evans	32
25	David Wilson	27
26	Richard Scott	38
27	Thomas Green	24
28	John King	31
29	James Lee	29
30	William Clark	33
31	Robert Adams	26
32	Thomas Baker	34
33	Charles Evans	23
34	David Wilson	36
35	Richard Scott	21
36	Thomas Green	37
37	John King	25
38	James Lee	30
39	William Clark	28
40	Robert Adams	35
41	Thomas Baker	22
42	Charles Evans	32
43	David Wilson	27
44	Richard Scott	38
45	Thomas Green	24
46	John King	31
47	James Lee	29
48	William Clark	33
49	Robert Adams	26
50	Thomas Baker	34
51	Charles Evans	23
52	David Wilson	36
53	Richard Scott	21
54	Thomas Green	37
55	John King	25
56	James Lee	30
57	William Clark	28
58	Robert Adams	35
59	Thomas Baker	22
60	Charles Evans	32
61	David Wilson	27
62	Richard Scott	38
63	Thomas Green	24
64	John King	31
65	James Lee	29
66	William Clark	33
67	Robert Adams	26
68	Thomas Baker	34
69	Charles Evans	23
70	David Wilson	36
71	Richard Scott	21
72	Thomas Green	37
73	John King	25
74	James Lee	30
75	William Clark	28
76	Robert Adams	35
77	Thomas Baker	22
78	Charles Evans	32
79	David Wilson	27
80	Richard Scott	38
81	Thomas Green	24
82	John King	31
83	James Lee	29
84	William Clark	33
85	Robert Adams	26
86	Thomas Baker	34
87	Charles Evans	23
88	David Wilson	36
89	Richard Scott	21
90	Thomas Green	37
91	John King	25
92	James Lee	30
93	William Clark	28
94	Robert Adams	35
95	Thomas Baker	22
96	Charles Evans	32
97	David Wilson	27
98	Richard Scott	38
99	Thomas Green	24
100	John King	31

$$\lg \sqrt[3]{\frac{\sqrt{2}}{16} + \sqrt{\frac{11}{3456}}} = \frac{\lg 0.14480529}{3} = \frac{2.1607845 - 3}{3} =$$

$$= 0.4202615 - 1 = \bar{1}.7202615$$

$$\sqrt[3]{\frac{\sqrt{2}}{16} + \sqrt{\frac{11}{3456}}} = 0.52512354$$

$$\lg \sqrt[3]{\frac{\sqrt{2}}{16} - \sqrt{\frac{11}{3456}}} = \frac{\lg 0.03197141}{3} = \frac{1.5047618 - 3}{3} =$$

$$= 0.5015873 - 1 = \bar{1}.5015873$$

$$\sqrt[3]{\frac{\sqrt{2}}{16} - \sqrt{\frac{11}{3456}}} = 0.31738566$$

$$\boxed{\cos \beta} = 0.52512354$$

$$+ \frac{0.31738566}{}$$

$$\boxed{\underline{\underline{0.84250920}}}$$

Los dos valores constantes de la ecuación cúbica (8), son imaginarios.

Del valor $\cos \beta = 0.84250920 \dots$ se deduce

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - 0.84250920^2} = 0.53868190 \dots (10)$$

7 de este

$$\beta = 32^\circ 35' 38.2''$$

1894-1895	1894-1895	1894-1895
1895-1896	1895-1896	1895-1896
1896-1897	1896-1897	1896-1897
1897-1898	1897-1898	1897-1898
1898-1899	1898-1899	1898-1899
1899-1900	1899-1900	1899-1900
1900-1901	1900-1901	1900-1901
1901-1902	1901-1902	1901-1902
1902-1903	1902-1903	1902-1903
1903-1904	1903-1904	1903-1904
1904-1905	1904-1905	1904-1905
1905-1906	1905-1906	1905-1906
1906-1907	1906-1907	1906-1907

Comprobación numérica de la raíz real

Hemos a comprobar numéricamente si la raíz real obtenida para la ecuación (8), la verifica. Dicha raíz es

$$x = \cos \beta = 0,84\ 25\ 09\ 20\ \dots$$

siendo la ecuación: $x^3 - \frac{1}{2}x - \frac{\sqrt{2}}{8} = 0$ o sea

$$x^3 - \frac{1}{2}x = \frac{\sqrt{2}}{8} = 0,17\ 67\ 76\ 70\ \dots$$

$$\begin{array}{r} x^2 = 0,84\ 25\ 09\ 20^2 = 0,70\ 98\ 14\ 00\ 10\ 00\ 00\ 00 \\ + \quad \quad \quad 7\ 75\ 10\ 84\ 64\ 00 \\ \hline 0,70\ 98\ 21\ 75\ 20\ 84\ 64\ 00 \end{array}$$

$$\begin{array}{r} x^3 = 0,70\ 98\ 21\ 75 \times 0,84\ 25\ 09\ 20 = \\ = 0,59\ 80\ 24\ 82\ 43\ 75\ 00\ 00 \\ + \quad \quad \quad 6\ 53\ 03\ 60\ 10\ 00 \\ \hline 0,59\ 80\ 31\ 35\ 47\ 35\ 10\ 00 \end{array}$$

$$\begin{array}{r} x^3 - \frac{1}{2}x = 0,59\ 80\ 31\ 35 \\ - \quad 0,42\ 12\ 54\ 60 \\ \hline 0,17\ 67\ 76\ 75\ \dots \approx \frac{\sqrt{2}}{8} = 0,17\ 67\ 76\ 70 \end{array}$$

cuyo resultado no comprueba la exactitud hasta la cifra 10^{-7} .

Comprobado el resultado numérico de $\cos \beta = 0,84\ 25\ 09\ 20\dots$ que nos confirma el de la fórmula (10)

$$\sin \beta = 0,53\ 86\ 81\ 90$$

llegaremos al resultado final del valor "m", según la fórmula (2)

$$m = \frac{l}{2 \sin \beta} = \frac{l}{2 \times 0,53\ 86\ 81\ 90} \quad l = \boxed{0,92\ 81\ 91\ 57\dots l}$$

Radio "a" de la esfera circumsrita

Aplicando la fórmula general [1], tendremos:

$$\begin{aligned} a &= \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - (0,92\ 81\ 91\ 57\dots l)^2}} = \frac{l}{2\sqrt{1 - 0,92\ 81\ 91\ 57^2}} \\ &= \frac{l}{2\sqrt{0,13\ 84\ 60\ 40\ 93\ 80\ 93\ 51}} \quad l = \frac{l}{2 \times 0,37\ 21\ 02\ 68} \quad l = \\ &= \boxed{1,34\ 37\ 15\ 13\dots l} \end{aligned}$$

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3], tendremos:

$$b = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{(1,34\ 37\ 15\ 13\dots l)^2 - \frac{l^2}{4}} = \sqrt{1,34\ 37\ 15\ 13^2 - 0,25} \quad l =$$

The first part of the report deals with the general situation of the country. It is a very interesting and informative study of the country's development. The second part of the report deals with the specific details of the country's development. It is a very detailed and thorough study of the country's development.

The third part of the report deals with the specific details of the country's development. It is a very detailed and thorough study of the country's development. The fourth part of the report deals with the specific details of the country's development. It is a very detailed and thorough study of the country's development.

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The eleventh part of the report deals with the specific details of the country's development. It is a very detailed and thorough study of the country's development. The twelfth part of the report deals with the specific details of the country's development. It is a very detailed and thorough study of the country's development.

$$\sqrt{1, 80 \ 55 \ 70 \ 35 \ 05 \ 90 \ 91 \ 69 - 0,25} \times l = \sqrt{1, 55 \ 55 \ 70 \ 35 \ 05 \ 90 \ 91 \ 69} \times l =$$

$$= \boxed{1, 24 \ 72 \ 25 \ 06 \dots l}$$

Radio " d_3 " de la circunferencia circunscrita a una cara triangular regular de lado " l ".

Se demuestra en Geometría es

$$\boxed{d_3 = \frac{\sqrt{3}}{3} l}$$

Radio " d_4 " de la circunferencia circunscrita a una cara cuadrada de lado " l ".

Se demuestra en Geometría es

$$\boxed{d_4 = \frac{\sqrt{2}}{2} l}$$

Radio " c_3 " de la esfera tangente a las caras triangulares regulares de lado " l ".

Aplicando la fórmula general [2], tendremos:

$$\boxed{c_3} = \sqrt{a^2 - (d_3)^2} = \sqrt{(1, 34 \ 37 \ 15 \ 13 \dots l)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} =$$

$$= \sqrt{1, 34 \ 37 \ 15 \ 13^2 - \frac{1}{3}} \times l = \sqrt{1, 80 \ 55 \ 70 \ 35 \ 05 \ 90 \ 91 \ 69 - \frac{1}{3}} \times l =$$

$$= \sqrt{1, 47 \ 22 \ 37 \ 01 \ 72 \ 57 \ 58 \ 36...} = \boxed{1, 21 \ 33 \ 57 \ 74... \ l}$$

Radio " c_4 " de la esfera tangente a las caras cuadradas de lado " l "

Aplicando la fórmula general [2], tendremos:

$$\boxed{c_4} = \sqrt{a^2 - (d_4)^2} = \sqrt{(1, 34 \ 37 \ 15 \ 13... \ l)^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} =$$

$$= \sqrt{1, 34 \ 37 \ 15 \ 13...^2 - \frac{1}{2}} \ l = \sqrt{1, 80 \ 55 \ 70 \ 35 \ 05 \ 90 \ 91 \ 69...} \ 0,5 \ l =$$

$$= \sqrt{1, 30 \ 55 \ 70 \ 35 \ 05 \ 90 \ 91 \ 69} \ l = \boxed{1, 14 \ 26 \ 15 \ 57... \ l}$$

Ángulo rectilíneo " α_3 " del diedro formado por una cara triangular, con el plano diametral del arquimediano, que pasa por una arista de aquella.

Se determina en función de su tangente, por la fórmula general [5]:

$$\frac{1}{\tan} \alpha_3 = \frac{2c_3}{\sqrt{4(d_3)^2 - l^2}} = \frac{2 \times 1, 21 \ 33 \ 57 \ 74... \ l}{\sqrt{4 \times \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \frac{2 \times 1, 21 \ 33 \ 57 \ 74}{\sqrt{4 \times \frac{3}{9} - 1}} =$$

$$= \frac{2 \times 1,21\ 33\ 57\ 74}{\sqrt{\frac{1}{3}}} = 2,42\ 67\ 15\ 18 \times \sqrt{3} = 4,20\ 31\ 94\ 51 \dots$$

$$\lg \operatorname{tg} \alpha_3 = 0,623\ 57\ 95$$

$$\alpha_3 = 76^\circ\ 37'\ 2,3''$$

Ángulo rectilíneo " α_4 " del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos, que pasa por una arista de aquélla.

Se determina en función de su tangente, por la fórmula general [6]

$$\operatorname{tg} \alpha_4 = \frac{2c_4}{\sqrt{4(d_4)^2 - l^2}} = \frac{2 \times 1,14\ 26\ 15\ 57 \dots l}{\sqrt{4 \times \left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} =$$

$$= \frac{2 \times 1,14\ 26\ 15\ 57 \dots}{\sqrt{2 - 1}} = 2,28\ 52\ 31\ 14 \dots$$

$$\lg \operatorname{tg} \alpha_4 = 0,358\ 9301$$

$$\alpha_4 = 66^\circ\ 21'\ 58,2''$$

Ángulo rectilíneo " φ_{3-3} " del diedro formado por dos caras triangulares

Aplicando la fórmula general [4]

$$\varphi_{3-3} = 2 \alpha_3 = 2 \times (76^\circ\ 37'\ 2,3'') = 153^\circ\ 14'\ 4,6''$$

--	--	--	--

The following is a list of the names of the persons who have been
 admitted to the office of the Secretary of the Board of Education
 since the last meeting of the Board, held on the 1st day of
 January, 1885. The names are given in alphabetical order, and
 the date of admission is given in parentheses.

Mr. J. H. Smith (Jan. 1, 1885)
 Mr. W. B. Jones (Jan. 1, 1885)
 Mr. C. D. Brown (Jan. 1, 1885)
 Mr. E. F. Green (Jan. 1, 1885)
 Mr. G. H. White (Jan. 1, 1885)
 Mr. I. J. Black (Jan. 1, 1885)
 Mr. K. L. Gray (Jan. 1, 1885)
 Mr. M. N. Hall (Jan. 1, 1885)
 Mr. O. P. King (Jan. 1, 1885)
 Mr. Q. R. Lee (Jan. 1, 1885)
 Mr. S. T. Young (Jan. 1, 1885)
 Mr. U. V. Wright (Jan. 1, 1885)
 Mr. X. Y. Scott (Jan. 1, 1885)
 Mr. Z. A. Adams (Jan. 1, 1885)

Ángulo rectilíneo " φ_{3-4} " del diedro formado por una cara triangular y una cuadrada

Aplicando la fórmula general [4]

$$\boxed{\varphi_{3-4}} = \alpha_3 + \alpha_4 = 76^\circ 37' 2,3'' + 66^\circ 31' 58,2''$$

$$= \boxed{142^\circ 59' 0,5''}$$



Área lateral "S" del arquimédiano

Se compone de 32 caras triangulares y 6 cuadradas, ambas de lado " l "; la superficie total será:

$$\boxed{S} = 32 \times \frac{\sqrt{3}}{4} l^2 + 6 l^2 = (8\sqrt{3} + 6) l^2 = \boxed{19, 25 \ 64 \ 06 \ 46 \dots l^2}$$

Volumen "V" del arquimédiano

Se compone de la suma de 32 pirámides de base triangular y altura c_3 , y de 6 pirámides de base cuadrada y altura c_4 ; su valor será

$$\boxed{V} = 32 \times \frac{\sqrt{3}}{4} l^2 \times \frac{1, 21 \ 33 \ 59 \ 74 \dots l}{3} + 6 \times l^2 \times \frac{1, 14 \ 26 \ 15 \ 57 \dots l}{3} =$$

$$= \boxed{7, 88 \ 74 \ 90 \ 48 \dots l^3}$$

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Desarrollo del cálculo anterior:

$$V = \left[\frac{32 \sqrt{3}}{4} \times \frac{1,21 \ 33 \ 57 \ 74}{3} + \frac{6}{3} \times 1,14 \ 26 \ 15 \ 57 \right] l^3$$

$$\frac{8 \sqrt{3}}{3} = 4,61 \ 88 \ 02 \ 15 ;$$

$$4,61 \ 88 \ 02 \ 15 \times 1,21 \ 33 \ 57 \ 74 = 5,60 \ 42 \ 59 \ 34$$

$$+ 2 \times 1,14 \ 26 \ 15 \ 57 = \frac{2,28 \ 52 \ 31 \ 14}{7,88 \ 94 \ 90 \ 48}$$

3. Topic _____

4. Introduction _____

5. Body _____

6. Conclusion _____

En el cuadro sinóptico que presentamos a continuación, resumimos los resultados anteriores

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\frac{l^2}{2\sqrt{l^2 - m^2}}$	1.34 37 15... l
b	$\sqrt{a^2 - \frac{l^2}{4}}$	1.24 72 25... l
c_3	$\sqrt{a^2 - (d_3)^2}$	1.21 33 58... l
c_4	$\sqrt{a^2 - (d_4)^2}$	1.14 26 16... l
d_3	$\frac{\sqrt{3}}{3} l$	0.57 73 50... l
d_4	$\frac{\sqrt{2}}{2} l$	0.70 71 07... l
m	$\frac{1}{2 \tan \beta} l$	0.72 81 92... l
α_3	$\tan^{-1} \alpha_3 = \frac{2c_3}{\sqrt{4(d_3)^2 - l^2}}$	$\tan^{-1} \alpha_3 = 2.42 67 15''$ $\alpha_3 = 76^\circ 37' 2.3''$
α_4	$\tan^{-1} \alpha_4 = \frac{2c_4}{\sqrt{4(d_4)^2 - l^2}}$	$\tan^{-1} \alpha_4 = 2.28 52 31''$ $\alpha_4 = 66^\circ 21' 58.2''$
ψ_{3-3}	$\alpha_3 + \alpha_3$	$\psi_{3-3} = 153^\circ 14' 4.6''$
ψ_{3-4}	$\alpha_3 + \beta_4$	$\psi_{3-4} = 142^\circ 59' 0.5''$
S	$(8\sqrt{3} + 6) l^2$	19.85 64 06... l^2
V	$\left(32 \times \frac{\sqrt{3}}{4} \times \frac{c_3}{3} + 6 \times \frac{c_4}{3}\right) l^2$	7.88 94 90... l^3
β	$\cos \beta = \sqrt{\frac{12}{16}} + \sqrt{\frac{11}{3456}} + \sqrt{\frac{12}{16}} - \sqrt{\frac{11}{3456}}$ $\sin \beta = \sqrt{1 - \cos^2 \beta}$	$\cos \beta = 0.84 25 09...$ $\beta = 32^\circ 35' 38.2''$ $\sin \beta = 0.53 86 82..$

Date	Description	Amount
	To Balance	100.00
	By Cash	50.00
	By Cash	25.00
	By Cash	15.00
	By Cash	10.00
	By Cash	5.00
	By Cash	5.00
	By Cash	5.00
	By Cash	5.00
	By Cash	5.00
	By Cash	5.00
	By Cash	5.00
	By Cash	5.00

Después del cálculo de magnitudes, vamos a proceder a la representación gráfica del arquimédico I, de lado dado.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotes complementarias cuyo cálculo justificaremos debidamente. Todas las magnitudes las obtendremos en función del lado l_1 del arquimédico, de 40,9 mm.

Calculemos previamente las siguientes magnitudes:

$$l_1 = 40,9 \text{ mm}$$

$$a = 1,343715 \times 40,9 = 55,0 \text{ mm}$$

$$b = 1,247225 \times 40,9 = 51,0 \text{ mm}$$

$$c_3 = 1,213358 \times 40,9 = 49,7 \text{ mm}$$

$$c_4 = 1,142616 \times 40,9 = 46,7 \text{ mm}$$

$$d_4 = 0,707107 \times 40,9 = 28,9 \text{ mm}$$

El orden de operaciones del trazado gráfico (lámin. 33) es el siguiente:

- 1° Situar el centro O, de coordenadas 72, 72, 85
- 2° Dibujar en I, II, y III la esfera circunscrita de radio $a = 55 \text{ mm}$.
- 3° Representar en I, II, y III la cara cuadrada 1-2-3-4, su puesto el poliedro colocado con dicha cara paralela a II y dos lados (1-4, 2-3) perpendiculares a I.



The first part of the paper deals with the general theory of the subject. It is divided into two main sections. The first section is devoted to the study of the properties of the function $f(x)$ and the second section is devoted to the study of the properties of the function $g(x)$.

In the first section, we shall study the properties of the function $f(x)$ and in the second section, we shall study the properties of the function $g(x)$.

Table 1	
x	$f(x)$
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20

The second part of the paper deals with the application of the theory to the study of the function $f(x)$. It is divided into two main sections. The first section is devoted to the study of the properties of the function $f(x)$ and the second section is devoted to the study of the properties of the function $g(x)$.

In the first section, we shall study the properties of the function $f(x)$ and in the second section, we shall study the properties of the function $g(x)$.

- 4° Obtener en I, II y III las proyecciones del vértice 5 de la cara contigua triangular de arista 1-4 (perpendicular a II), por giro alrededor de 1-4 hasta colocar el vértice 5 sobre la esfera circunscrita. Para ello se hará centro en 1_I , un radio igual a la altura de la cara 1-4-5, y se trazará un arco que corte en 5_I a la esfera circunscrita.
- 5° Determinar las proyecciones en II y III de dicho vértice 5, y seguidamente en I, II y III las de los vértices 6, 7 y 8. (Los vértices 5, 6, 7, y 8 son los de un cuadrado paralelo al II).
- 6° Determinar las proyecciones en I y II de la arista 2-11, por giro de la misma alrededor del eje perpendicular a II que pase por el centro O, hasta colocarla paralela a I. (Los puntos 2_I y 11_I están sobre la esfera circunscrita y el segmento 2_I-11_I es la arista. Obsérvese también que la arista 2-11 en la proyección II, pasa por O_{II}).
- 7° Determinar la proyección en III del punto 11, y seguidamente en I, II y III las de los vértices 9, 10, 12. (Los vértices 9, 10, 11 y 12 son los de un cuadrado paralelo a II).
- 8° Trazar en II el eje "h" que forma con el 5-7 (paralelo este último al X) un ángulo $\varepsilon = 16^\circ 28' 31''$ ($\tan \varepsilon = 0.29559$).
- 9° Los restantes vértices del poliedro 13 al 24 son simétricos en II con respecto a dicho eje "h", por

No.	Date	Page
<p>1</p>	<p>2000</p>	<p>1</p>

lo que si situación en II es inmediata. Uniendo debidamente todos los vértices y estudiando la visibilidad de las aristas correspondientes, podremos representar en II la proyección total del poliedro.

- 10º Para completar la representación en I y III del poliedro, basta tener en cuenta que los vértices 13 al 16 están situados sobre un plano paralelo al de los vértices 9 al 12 y equidistantes del diametral paralelo a II. Igualmente ocurre con los vértices 17 al 20 con respecto a los 5 al 8, así como los 21 al 24 con respecto a los 1 al 4.

Como comprobación al trazado gráfico dado anteriormente vamos a determinar analíticamente las siguientes cotas complementarias que darán mayor exactitud a dicho trazado.

Altura "n" de una cara triangular

$$n = \frac{\sqrt{3}}{2} l = 0.866025... l$$

Para el caso del dibujo será $n = 0.8660254 \times 40.9 = 35.4 \text{ m.}$

Distancia "g" de los vértices 5 al 8 al plano de la cara cuadrada 1-2-3-4, y de los 17 al 20 a la cara

drada opuesta 20 al 211

Se obtiene proyectando la altura "h" sobre el plano III;
el ángulo de proyección es de

$$\varphi_{3.4} - \frac{\pi}{2} = 142^{\circ} 59' 0,5'' - 90^{\circ} = 52^{\circ} 59' 0,5''$$

$$\boxed{g_1} = h \times \cos 52^{\circ} 59' 0,5'' = \frac{\sqrt{3}}{2} \times \cos 52^{\circ} 59' 0,5'' \times \ell =$$

$$= \boxed{0,52 \ 13 \ 86 \ 5 \times \ell}$$



Desarrollo del cálculo anterior:

$\frac{1}{2} \log 3 = \frac{1}{2} \times 0,477 \ 12 \ 13 =$	0,238 56 07
$+ \lg. \cos 52^{\circ} 59' 0,5'' =$	7,779 62 92 +
	0,018 18 99
$- \lg 2 =$	0,301 03 00 -
$\lg \boxed{0,52 \ 13 \ 86 \ 5} =$	<u>7,717 15 99</u>

Para el caso del dibujo será: $g_1 = 0,52 \ 13 \ 86 \ 5 \times 40,9 = 21,3 \text{ mm.}$

Distancia "f₁" entre los dos planos paralelos a II, que contienen los vértices 5 al 8, y 17 al 20 respectivamente.

Se obtiene por diferencias de alturas "c₄" y "g₁", ya calculadas;

$$\boxed{f_1} = 2 (c_4 - g_1) = 2 \times (1,14 \ 26 \ 15 \ 6 - 0,52 \ 13 \ 86 \ 5) \times \ell =$$

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$$= \boxed{1, 24 \ 24 \ 58 \ 2 \dots l}$$

Para el caso particular del dibujo, será: $l_1 = 1, 24 \ 24 \ 58 \ 2 \times 40,7 = 50,8 \text{ mm.}$

Distancia "g" de los vértices 9 al 12 al plano de la cara cuadrada 1-2-3-4, y de los 17 al 20 a la cara cuadrada opuesta 20 al 24.

Se obtiene girando previamente la arista 2-11 hasta colocarla paralelamente al plano I, y proyectarla seguidamente sobre el plano III.

Para calcularla, determinemos previamente el ángulo de proyección de dicha arista.

Si consideramos el plano diametral que pasa por 2-11 y el centro O del poliedro, dicho plano pasará a su vez por el centro de la cara cuadrada 1 al 4; uniendo ambos centros tendremos determinado el eje de giro de dicha arista 2-11, el cual será perpendicular a II; los puntos 2 y 11 en II, pasarán a ocupar, después del giro, las posiciones 2_I y 11_I en I, y la arista $2_I - 11_I$ estará proyectada en I en su verdadera magnitud.

Uniendo a continuación 2_I y 11_I con el centro O del poliedro, se nos formará el triángulo isósceles $O - 2_I - 11_I$, de base "l", altura "b" y lado "a" (fig. 2)

1. The first part of the report deals with the general situation of the country. It is a very interesting and informative study of the country's development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's development.

2. The second part of the report deals with the economic situation of the country. It is a very interesting and informative study of the country's economic development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's economic development.

3. The third part of the report deals with the social situation of the country. It is a very interesting and informative study of the country's social development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's social development.

4. The fourth part of the report deals with the political situation of the country. It is a very interesting and informative study of the country's political development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's political development.

5. The fifth part of the report deals with the cultural situation of the country. It is a very interesting and informative study of the country's cultural development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's cultural development.

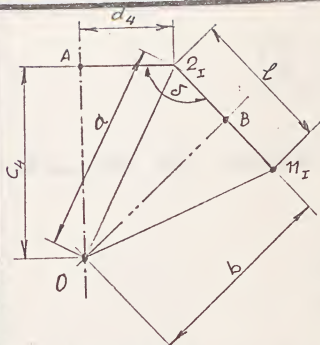


Figura 2

Este triángulo será contiguo al $O \cdot A \cdot 2_I$, rectángulo en A, de catetos " c_4 ", " d_4 " e hipotenusa " a ".

El ángulo $\widehat{A \cdot 2_I \cdot n_I}$ es el que forma la arista " l " con el plano de la cara cuadrada 1 al 4, siendo

$$S = \widehat{A \cdot 2_I \cdot n_I} = \widehat{A \cdot 2_I \cdot O} + \widehat{O \cdot 2_I \cdot B} \quad \text{de donde}$$

$$\cos(\widehat{A \cdot 2_I \cdot O}) = \frac{d_4}{a} = \frac{\frac{\sqrt{2}}{2} l}{\frac{l^2}{2\sqrt{l^2 - m^2}}} = \frac{0.70710678...}{1.34371513...} = 0.52623265...$$

$$\angle A \cdot 2_I \cdot O = 58^\circ 14' 55.5''$$

Desarrollo del cálculo anterior:

$$\begin{aligned} & \angle 0.70710678... = \overline{7, 8494850} \\ - & \angle 1.34371513... = \underline{-0, 1283072} \\ & \angle \cos(\widehat{A \cdot 2_I \cdot O}) = \overline{7, 7211779} \end{aligned}$$

$$\angle A \cdot 2_I \cdot O = 58^\circ 14' 55.5''$$

y por otra parte

$$\begin{aligned} \cos(\widehat{O \cdot 2_I \cdot B}) &= \frac{\frac{l}{2}}{a} = \frac{l}{2a} = \frac{l}{2 \times \frac{l^2}{2\sqrt{l^2 - m^2}}} = \frac{1}{2 \times 1.34371513...} \\ &= \frac{1}{2.68743026} = 0.37210268... \end{aligned}$$

$$\angle O-2_I-B = 68^{\circ} 9' 16,7''$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \lg 1 &= 0,000\ 00\ 00 \\ \lg 2 &= 0,301\ 03\ 00 \\ + \lg 1,34\ 37\ 15\ 13... &= 0,128\ 30\ 72 = -0,429\ 33\ 72 \\ \lg \cos(\widehat{O-2_I-B}) &= \underline{\underline{7,570\ 66\ 28}} \end{aligned}$$

$$\angle O-2_I-B = 68^{\circ} 9' 16,7''$$

y por lo tanto tendremos que

$$\begin{aligned} \boxed{\angle} &= \angle A-2_I-11_I = 58^{\circ} 14' 55,5'' + 68^{\circ} 9' 16,7'' = \\ &= \boxed{126^{\circ} 24' 12,2''} \end{aligned}$$

Si proyectamos la arista 2_I-11_I sobre el plano III, el ángulo de proyección será:

$$126^{\circ} 24' 12,2'' - 90^{\circ} = 36^{\circ} 24' 12,2''$$

por lo que finalmente tendremos:

$$\boxed{g_2} = l \cos 36^{\circ} 24' 12,2'' = \boxed{0,80\ 48\ 58\ 89.... l}$$

Desarrollo del cálculo anterior:

$$\lg \cos 36^{\circ} 24' 12,2'' = 7,905\ 71\ 97$$

$$\boxed{\cos 36^{\circ} 24' 12,2''} = \text{Antilog } 7,905\ 71\ 97 = \boxed{0,80\ 48\ 58\ 89...}$$

Dear Sir,

I have the honor to acknowledge the receipt of your letter of the _____ dated _____ regarding _____.

I am sorry to hear that _____.

I have discussed this matter with _____ and we have decided to _____.

I am enclosing herewith _____.

I am sure that you will be satisfied with the outcome of this matter.

Yours faithfully,

Para el caso particular del dibujo, será:

$$j = 0,80485889 \times 40,9 = 32,9 \text{ mm.}$$

Distancia "f₂" entre los dos planos paralelos a II, que contienen los vértices 9 al 12 y 13 al 16 respectivamente.

Se obtiene por diferencia de las alturas "c₄" y "j", ya calculadas

$$\begin{aligned} f_2 &= 2(c_4 - j) = 2 \times (1,1426156... - 0,80485889...) \ell = \\ &= \boxed{0,6755134... \ell} \end{aligned}$$

Para el caso particular del dibujo, será: $i = 0,6755134 \times 40,9 = 27,6 \text{ mm.}$

Radio "r₁" de la circunferencia circunscrita a los cuadrados de vértices 5 al 8 y 17 al 20.

Esta' representado en su verdadera magnitud, en el plano II, por el segmento 5-O, suma del 5-B y del B-O. Sus valores son:

$$\left. \begin{aligned} 5-B &= n \times \text{sen } 52^\circ 59' 0,5'' \quad (\text{ver cálculo de "j"}) \\ B-O &= \frac{\ell}{2} \end{aligned} \right\} \text{ de donde}$$

$$r_1 = n \text{ sen } 52^\circ 59' 0,5'' + \frac{\ell}{2} = \left(\frac{\sqrt{3}}{2} \times \text{sen } 52^\circ 59' 0,5'' + \frac{1}{2} \right) \times \ell =$$

--	--	--

The first of these is the fact that the
 number of people who are employed in the
 service of the government is increasing
 rapidly. This is due to the fact that the
 government is expanding its activities in
 many fields, and is therefore requiring
 more and more people to work for it.
 The second fact is that the number of
 people who are employed in the service of
 the government is increasing rapidly.
 This is due to the fact that the
 government is expanding its activities in
 many fields, and is therefore requiring
 more and more people to work for it.
 The third fact is that the number of
 people who are employed in the service of
 the government is increasing rapidly.
 This is due to the fact that the
 government is expanding its activities in
 many fields, and is therefore requiring
 more and more people to work for it.
 The fourth fact is that the number of
 people who are employed in the service of
 the government is increasing rapidly.
 This is due to the fact that the
 government is expanding its activities in
 many fields, and is therefore requiring
 more and more people to work for it.
 The fifth fact is that the number of
 people who are employed in the service of
 the government is increasing rapidly.
 This is due to the fact that the
 government is expanding its activities in
 many fields, and is therefore requiring
 more and more people to work for it.

--	--	--

$$= \boxed{1.19\ 14\ 88\ 4 \dots l}$$

Desarrollo del cálculo anterior:

$$\frac{1}{2} l_3 = \frac{1}{2} \times 0,477\ 12\ 13 = 0,238\ 56\ 07$$

$$+ l_2 \text{ sen } 52^\circ 59' 0,5'' = \frac{7,902\ 25\ 42}{0,140\ 81\ 49}$$

$$- l_2 = -0,301\ 03\ 00$$

$$\text{Antilog } 7,839\ 78\ 49 = 0,69\ 14\ 88\ 4 \dots$$

$$\boxed{r_1} = (0,69\ 14\ 88\ 4 \dots + 0,5) l = \boxed{1.19\ 14\ 88\ 4 \dots l}$$

Para el caso del dibujo, será: $r_1 = 1.19\ 14\ 88\ 4 \times 40,9 = 48,7_{mm}$

Radio " r_2 " de la circunferencia circunscrita a los cuadrados de vértices 9 al 12 y 13 al 16.

Está representado en su verdadera magnitud, en el plano II, por el segmento 11-0, suma del 0-2 y del 2-11. Sus valores son:

$$0-2 = d_4 \times l = \frac{\sqrt{2}}{2} l$$

$$2-11 = l \text{ sen } 36^\circ 24' 12,2'' \text{ (ver cálculo de "j")} \left. \vphantom{\frac{\sqrt{2}}{2} l} \right\} \text{ de donde}$$

$$\boxed{r_2} = \left(\frac{\sqrt{2}}{2} + \text{sen } 36^\circ 24' 12,2'' \right) l = (0,70\ 71\ 06\ 8 + 0,59\ 34\ 66\ 4) l$$

$$= \boxed{1.30\ 05\ 73\ 9 \dots l}$$

EL

30-11-72

1. 1. 1. 1. 1. 1.

2. 2. 2. 2. 2. 2.

3. 3. 3. 3. 3. 3.

4. 4. 4. 4. 4. 4.

5. 5. 5. 5. 5. 5.

6. 6. 6. 6. 6. 6.

7. 7. 7. 7. 7. 7.

8. 8. 8. 8. 8. 8.

9. 9. 9. 9. 9. 9.

10. 10. 10. 10. 10. 10.

11. 11. 11. 11. 11. 11.

12. 12. 12. 12. 12. 12.

13. 13. 13. 13. 13. 13.

14. 14. 14. 14. 14. 14.

15. 15. 15. 15. 15. 15.

16. 16. 16. 16. 16. 16.

Desarrollo del cálculo anterior:

$$\frac{1}{2} \log 2 = \frac{1}{2} \times 0,301\ 03\ 00 = 0,150\ 51\ 50$$

$$- \log 2 = -0,301\ 03\ 00$$

$$\text{Antilog } 7,849\ 48\ 50 = 0,70\ 71\ 06\ 8...$$

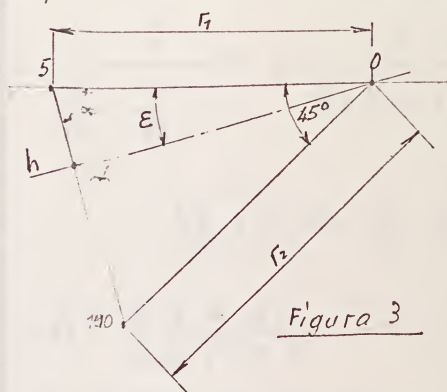
$$\log \tan 36^\circ\ 24'\ 12,2'' = 7,77\ 33\ 96\ 2$$

$$\text{Antilog } 7,77\ 33\ 96\ 2... = 0,59\ 34\ 66\ 4...$$

$$\boxed{r_2 = (0,70\ 71\ 06\ 8... + 0,59\ 34\ 66\ 4...) \ell = 1,30\ 05\ 73\ 2... \ell}$$

Para el caso del dibujo, será: $r_2 = 1,30\ 05\ 73\ 2 \times 40,9 = 53,2\ \text{mm}$

Ángulo "ε" que forma el eje de simetría "h", en la proyección II, con el eje paralelo a "x".



Refiriéndonos a la proyección II, el triángulo 5-0-10 (fig. 3), tiene los valores ya calculados $5-0 = r_1$, $10-0 = r_2$, y el ángulo que forman estos lados, es de 45° .

La altura OA de este triángulo, correspondiente a "0", es el eje "h" buscado, que forma el ángulo "ε" con el lado 5-0.

1. The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations.

2. In the second part, we consider the case of a linear differential equation. It is shown that the problem is solvable in this case.

3. In the third part, we consider the case of a nonlinear differential equation. It is shown that the problem is solvable in this case.

4. In the fourth part, we consider the case of a system of differential equations. It is shown that the problem is solvable in this case.

5. In the fifth part, we consider the case of a partial differential equation. It is shown that the problem is solvable in this case.

6. In the sixth part, we consider the case of a boundary value problem. It is shown that the problem is solvable in this case.

En la resolución trigonométrica de triángulos oblicuángulos, cuando se conocen dos lados y el ángulo comprendido, se obtiene otro de sus ángulos por la fórmula

$$\tan \alpha = \frac{a \operatorname{sen} \gamma}{b - a \cos \gamma}$$

que aplicada al caso particular de la fig. 3, haciendo

$$a = r_2; \quad b = r_1; \quad \gamma = 45^\circ \quad y \quad \alpha = \widehat{0-5-10}, \quad y$$

$$\text{teniendo en cuenta que } \operatorname{sen} 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$$

y que $\tan \alpha = \operatorname{ctg} \varepsilon$, tendremos

$$\begin{aligned} \tan \alpha = \operatorname{ctg} \varepsilon &= \frac{r_2 \times \frac{\sqrt{2}}{2}}{r_1 - r_2 \times \frac{\sqrt{2}}{2}} = \frac{1}{\frac{r_1}{r_2 \times \frac{\sqrt{2}}{2}} - 1} = \\ &= \frac{1}{\frac{r_1}{r_2} \times \frac{2}{\sqrt{2}} - 1} = \frac{1}{\sqrt{2} \times \frac{r_1}{r_2} - 1} \quad y \text{ de aquí:} \end{aligned}$$

$$\begin{aligned} \boxed{\operatorname{ctg} \varepsilon} &= \sqrt{2} \times \frac{r_1}{r_2} - 1 = \sqrt{2} \times \frac{1.19 \ 14 \ 88 \ 4 \dots 6}{1.30 \ 05 \ 73 \ 2 \dots 1} - 1 = \\ &= \sqrt{2} \times \frac{1.19 \ 14 \ 88 \ 4}{1.30 \ 05 \ 73 \ 2} - 1 = \boxed{0.29 \ 55 \ 97 \ 3 \dots} \end{aligned}$$

Desarrollo del cálculo anterior:

$$\boxed{\varepsilon = 16^\circ \ 28' \ 3.7''}$$

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

It is shown that the function $f(x)$ is continuous and differentiable on the interval $(0, \infty)$. The derivative of the function is given by the formula

$$f'(x) = -\frac{f(x)}{x}$$

From this it follows that the function $f(x)$ satisfies the differential equation

$$x f'(x) + f(x) = 0$$

The general solution of this equation is given by the formula

$$f(x) = \frac{C}{x}$$

$$f(x) = \frac{1}{x}$$

$$\frac{1}{2} l_1 \cdot 2 = \frac{1}{2} \times 0,301\ 03\ 00 = 0,150\ 51\ 50$$

$$+ l_1 \cdot 1,19\ 14\ 88\ 4 = 0,076\ 08\ 98 +$$

$$0,226\ 60\ 48$$

$$- l_1 \cdot 1,30\ 05\ 73\ 2 = 0,114\ 13\ 48 -$$

$$\text{Antilog } 0,112\ 47\ 00 = 1,29\ 55\ 97\ 3...$$

$$\boxed{k_1 \varepsilon = 1,29\ 55\ 97\ 3 - 1 = 0,29\ 55\ 97\ 3}$$

$$l_1 k_1 \varepsilon = 7,470\ 70\ 04$$

$$\boxed{\varepsilon = 16^\circ\ 28'\ 3,1''}$$

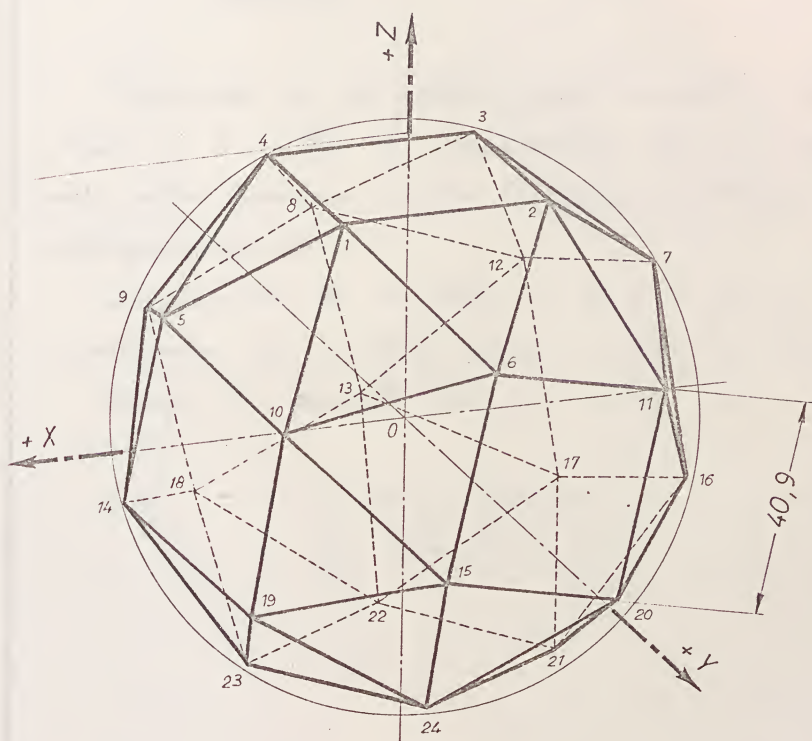
CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
n	$\frac{\sqrt{3}}{2} l$	$0,86\ 60\ 25... l$
f_1	$2 (C_4 - g_1)$	$1,24\ 24\ 58... l$
g_1	$\frac{\sqrt{3}}{2} \times \cos 52^\circ 59' 0,5'' l$	$0,52\ 13\ 87... l$
f_2	$2 (C_4 - g_2)$	$0,67\ 55\ 13... l$
g_2	$\cos 36^\circ 24' 12,2'' l$	$0,80\ 48\ 59... l$
f_1	$(\frac{\sqrt{3}}{2} \sin 52^\circ 59' 0,5'' + \frac{1}{2}) l$	$1,19\ 14\ 88... l$
f_2	$(\frac{\sqrt{2}}{2} + \sin 36^\circ 24' 12,2'') l$	$1,30\ 05\ 73... l$
ε	$k_1 \varepsilon = \sqrt{2} \cdot \frac{f_1}{f_2} - 1$	$k_1 \varepsilon = 0,29\ 55\ 97$ $\varepsilon = 16^\circ\ 28'\ 3,1''$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 32 triángulos equiláteros y 6 cuadrados, de lado 40.9 mm. de forma que en cada vértice concurren 4 triángulos y 1 cuadrado

55,0



Arquimediano I



Fig. 1. Dodecahedron

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimedeano II, en el que en cada vértice concurren cuatro triángulos equiláteros y un pentágono regular.

La longitud de su lado es de 25,5 mm, y las coordenadas de su centro O, son: $O(72, 72, 85)$ mm.

Dibujar en formato A3V y a escala 1:1.

DATOS:

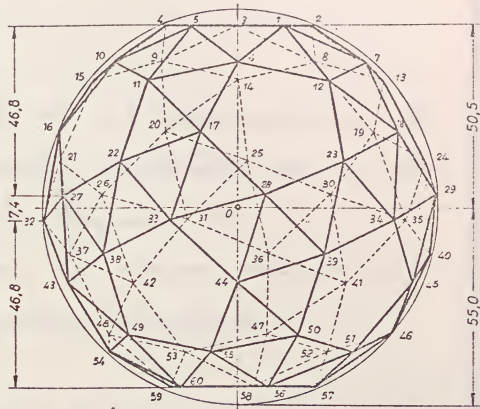
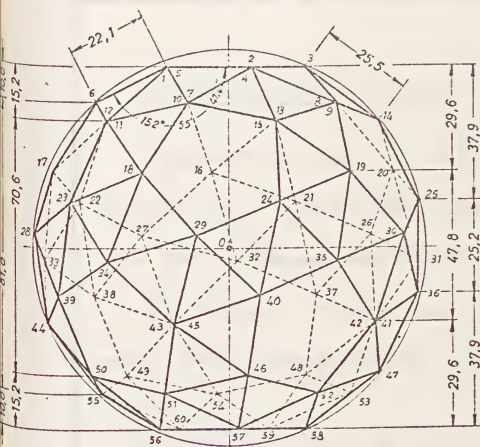
$O(72, 72, 85)$ mm

$l_{II} = 25,5$ mm.

I

+ Z

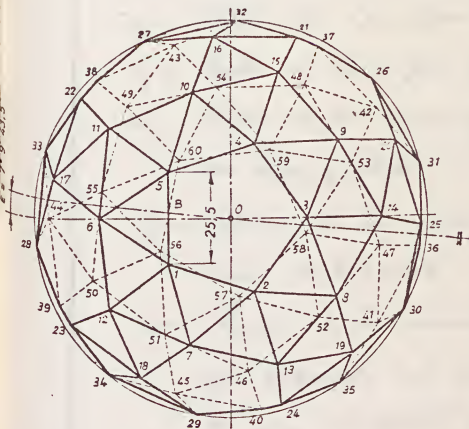
III



+ X

O

+ Y



ARQUIMEDIANO II

Número de caras triangulares..... $C_3 = 80$
 Número de caras pentagonales..... $C_5 = 12$
 Número de vértices..... $V = 60$
 Número de aristas..... $A = 150$
 Número de caras de un ángulo sólido: $4C_3 + 1C_5$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III el Arquimediano II, en el que en cada vértice concurren cuatro triángulos equiláteros y un pentágono regular.

La longitud de su lado es de 25,5 milímetros y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

+ Y

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					
Alumno:					
Escala	Arquimediano II				Lámina 34
1:1					Curso 19 - 19

II

CONSIDERACIONES PREVIAS

Seguiremos, en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el estudio del "Arquimediano I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes generales siguientes:

l = Arista del Arquimediano II (dato del problema)

a = Radio de la esfera circunscrita

b = Radio de la esfera tangente a las aristas

c_3 = Radio de la esfera tangente a las caras triangulares

c_5 = Radio de la esfera tangente a las caras pentagonales

d_3 = Radio de la circunferencia circunscrita a una cara triangular

d_5 = Radio de la circunferencia circunscrita a una cara pentagonal

m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

α_3 = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimedianos, que pasa por una arista de aquélla.

α_5 = Ángulo rectilíneo del diedro formado por una cara pentagonal, con el plano diametral del arquimedianos.

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mediano, que pasa por una arista de aquélla.

φ_{3-3} = Ángulo rectilíneo del diedro formado por dos caras triangulares.

φ_{3-5} = Ángulo rectilíneo del diedro formado por una cara triangular y otra pentagonal

S = Superficie

V = Volumen.

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimediano, nos indica que tiene 80 caras regulares triangulares, y 12 caras regulares pentagonales; 60 vértices y 150 aristas.

En cada vértice concurren 4 caras triangulares, 1 cara pentagonal y, por consiguiente, 5 aristas del mismo.

Así pues, tendremos que

$$\text{Arquimediano II } (4 P_3 + 1 P_5); C_3 = 80; C_5 = 12; V = 60; A = 150$$

Cálculo de sus magnitudes

Arista "l" del arquimediano

Dato del ejercicio



THE [illegible] OF [illegible]

[illegible]

[illegible text block]

[illegible text block]

[illegible text block]



[illegible text block]

[illegible text block]

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las cinco aristas de un ángulo poliedro.

Este polígono será un pentágono irregular (concurrerán en el vértice 4 triángulos equiláteros y un pentágono) formado por cuatro lados iguales y uno desigual.

Los cuatro lados iguales tienen una longitud igual a la arista "l" (tercer lado de una cara triangular regular) y el quinto tendrá una longitud igual a la diagonal de un pentágono regular de lado "l".

Su valor será pues:

$$AB = a = l \frac{\sqrt{5} + 1}{2}$$

y la relación $\frac{a}{l} = \frac{\sqrt{5} + 1}{2}$

Siguiendo el mismo proceso de cálculo desarrollado en la lámina 33 para la determinación de "m" en el Arquimedianos I, y refiriéndonos a la misma figura 1, tendremos que

$$m = \frac{l}{2 \cos \beta} \quad (1)$$

en la que $\cos \beta$ se deduce en función de $\cos \beta$, que es a su vez la solución real de la ecuación cúbica

$$8 \cos^3 \beta - 4 \cos \beta = \frac{a}{l} = \frac{\sqrt{5} + 1}{2} \quad (2)$$

of the Royal Society of London, in the year 1841, at the meeting of the Society, held on the 15th of January, 1841, at the Royal Society, London.

The following is a list of the names of the members of the Royal Society of London, who were present at the meeting of the Society, held on the 15th of January, 1841, at the Royal Society, London.

JOHN TYNDALL, Esq., F.R.S., Secy.

JOHN TYNDALL, Esq., F.R.S., Secy.

JOHN TYNDALL, Esq., F.R.S., Secy.

JOHN TYNDALL, Esq., F.R.S., Secy.

para el caso particular que nos ocupa.

Haciendo en (2) $\cos \beta = x$, tendremos

$$8x^2 - 4x = \frac{\sqrt{5}+1}{2} \quad " \quad 8x^3 - 4x = \frac{\sqrt{5}+1}{2} = 0$$

$$x^3 - \frac{1}{2}x - \frac{\sqrt{5}+1}{16} = 0 \quad (3)$$

que se puede transformar, haciendo $p = -\frac{1}{2}$ y

$q = -\frac{\sqrt{5}+1}{16}$, en la general

$$x^3 + px + q = 0$$

La fórmula de Cardano nos permite obtener "z" real, siempre que se verifique que

$$R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 > 0.$$

lo cual sucede en este caso, ya que

$$R = \left(-\frac{\sqrt{5}+1}{16} : 2\right)^2 + \left(-\frac{1}{2} : 3\right)^3 = \frac{3+\sqrt{5}}{512} - \frac{1}{216} = \frac{17+27\sqrt{5}}{13824} > 0$$

Desarrollo del cálculo anterior:

$$R = \left(-\frac{\sqrt{5}+1}{16} : 2\right)^2 + \left(-\frac{1}{2} : 3\right)^3 = \frac{(\sqrt{5}+1)^2}{32^2} - \frac{1}{2^3 \times 3^3} = \frac{5+1+2\sqrt{5}}{32^2} - \frac{1}{2^3 \times 3^3} =$$

$$= \frac{6+2\sqrt{5}}{32^2} - \frac{1}{2^3 \times 3^3} = \frac{3+\sqrt{5}}{16 \times 32} - \frac{1}{2^3 \times 3^3} = \frac{3+\sqrt{5}}{2^9} - \frac{1}{2^3 \times 3^3} =$$

$$= \frac{2^3 \times 3^4 + 2^3 \times 3^3 \times \sqrt{5} - 2^9}{2^{12} \times 3^3} = \frac{3^4 + 3^3 \sqrt{5} - 2^6}{2^9 \times 3^3} = \frac{17+27\sqrt{5}}{13824} > 0$$

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined on the interval $[0, 1]$. It is shown that $f(x)$ is continuous and differentiable on this interval.

In the second part, we consider the problem of finding the maximum and minimum values of the function $f(x)$ on the interval $[0, 1]$. It is shown that the function has a local maximum at $x = \frac{1}{2}$ and a local minimum at $x = 0$ and $x = 1$.

The third part of the paper is devoted to the study of the properties of the function $f(x)$ on the interval $[0, 1]$. It is shown that $f(x)$ is continuous and differentiable on this interval.

In the fourth part, we consider the problem of finding the maximum and minimum values of the function $f(x)$ on the interval $[0, 1]$. It is shown that the function has a local maximum at $x = \frac{1}{2}$ and a local minimum at $x = 0$ and $x = 1$.

The fifth part of the paper is devoted to the study of the properties of the function $f(x)$ on the interval $[0, 1]$. It is shown that $f(x)$ is continuous and differentiable on this interval.

In the sixth part, we consider the problem of finding the maximum and minimum values of the function $f(x)$ on the interval $[0, 1]$. It is shown that the function has a local maximum at $x = \frac{1}{2}$ and a local minimum at $x = 0$ and $x = 1$.

7 por consiguiente:

$$\begin{aligned} \boxed{z} &= \sqrt[3]{-\frac{9}{2} + \sqrt{R}} + \sqrt[3]{-\frac{9}{2} - \sqrt{R}} = \sqrt[3]{-\left(-\frac{\sqrt{5}+1}{16} : 2\right) + \sqrt{\frac{17+27\sqrt{5}}{13824}}} + \\ &+ \sqrt[3]{-\left(-\frac{\sqrt{5}+1}{16} : 2\right) - \sqrt{\frac{17+27\sqrt{5}}{13824}}} = \sqrt[3]{\frac{\sqrt{5}+1}{32} + \sqrt{\frac{17+27\sqrt{5}}{13824}}} + \sqrt[3]{\frac{\sqrt{5}+1}{32} - \sqrt{\frac{17+27\sqrt{5}}{13824}}} = \\ &= 0,56\ 03\ 44\ 87\dots + 0,29\ 74\ 35\ 82\dots = \boxed{0,85\ 77\ 80\ 69\dots} = \cos \beta \\ &\quad 0,85\ 77\ 80\ 74\ 99\dots \end{aligned}$$

Desarrollo del cálculo anterior:

$$\frac{\sqrt{5}+1}{32} = \frac{3,23\ 60\ 67\ 97\ 74\ 99\ 79\dots}{32} = 0,10\ 11\ 27\ 12\ 42\ 96\ 87\dots$$

$$\frac{17+27\sqrt{5}}{13824} = \frac{17+60,37\ 38\ 35\ 39\ 24\ 94\ 33\dots}{13824} = 0,00\ 55\ 97\ 06\ 56\ 38\ 92\dots$$

$$\sqrt{\frac{17+27\sqrt{5}}{13824}} = \sqrt{0,00\ 55\ 97\ 06\ 56\ 38\ 92\dots} = 0,07\ 48\ 13\ 54\dots$$

$$z = \sqrt[3]{0,17\ 59\ 40\ 66\dots} + \sqrt[3]{0,02\ 63\ 13\ 58\dots}$$

$$\frac{1}{3} \frac{1}{3} 0,17\ 59\ 40\ 66\dots = \bar{1},245\ 3662 = 2,245\ 3662 - 3$$

$$\frac{1}{3} \frac{1}{3} 0,17\ 59\ 40\ 66\dots = \frac{1}{3} \times (2,245\ 3662 - 3) = 0,748\ 4554 - 1 = \bar{1},748\ 4554$$

$$\sqrt[3]{0,17\ 59\ 40\ 66\dots} = \text{Antilog. } \bar{1},748\ 4554 = 0,56\ 03\ 44\ 87\dots$$

$$\frac{1}{3} \frac{1}{3} 0,02\ 63\ 13\ 58\dots = \bar{2},420\ 18\ 00 = 1,420\ 18\ 00 - 3$$

$$\frac{1}{3} \frac{1}{3} 0,02\ 63\ 13\ 58\dots = \frac{1}{3} (1,420\ 18\ 00 - 3) = 0,473\ 3933 - 1 = \bar{1},473\ 3933$$

$$\sqrt[3]{0,02\ 63\ 13\ 52} = \text{Analog. } \bar{7},473\ 3933 = 0,29\ 74\ 35\ 82...$$

$$\cos \beta = \boxed{x} = 0,56\ 03\ 44\ 87 + 0,29\ 74\ 35\ 82 = \boxed{0,85\ 77\ 80\ 69...}$$

$$\lg \cos \beta = \lg 0,85\ 77\ 80\ 69 = \bar{7},933\ 37\ 63 \quad \beta = 30^\circ 55' 54,1''$$

Los dos valores constantes de la ecuación cúbica, son imaginarios.

Del valor $\cos \beta = 0,85\ 77\ 80\ 69...$ se deduce

$$\sin \beta = \sin 30^\circ 55' 54,1'' = \sqrt{1 - 0,85\ 77\ 80\ 69^2} = 0,51\ 40\ 15\ 84^{\vee}$$

0,51 40 15 74 48

$$\text{con } \beta = 0,51\ 40\ 15\ 84 \quad (4)$$

Comprobación numérica de la raíz real

Vamos a comprobar numéricamente si la raíz real obtenida para la ecuación (3), la verifica.

Dicha raíz es

$$x = \cos \beta = 0,85\ 77\ 80\ 69$$

0,85 77 80 74 99

siendo la ecuación:

$$x^3 - \frac{1}{2}x - \frac{\sqrt{5}+1}{16} = 0$$

1911. Vol. 10. No. 1. 1-100.

CONTENTS.

1. The Effect of the ...

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Received of the Treasurer of the University of Chicago
the sum of \$100.00
for the year ending June 30, 1911

By the Treasurer,
J. H. ...

Witness my hand and seal this 1st day of July, 1911

At Chicago, Illinois

Very truly yours,
J. H. ...

By the Treasurer,
J. H. ...

THE UNIVERSITY OF CHICAGO

Radio "a" de la esfera circunscrita

Aplicando la fórmula general [1] (ver lám. 33)

$$\begin{aligned}
 [a] &= \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - (0,97\ 27\ 32\ 67... l)^2}} = \frac{1}{2\sqrt{1 - (0,97\ 27\ 32\ 67...)^2}} l = \\
 &= \frac{1}{2\sqrt{1 - 0,94\ 62\ 08\ 84\ 72\ 85\ 32\ 89}} l = \frac{1}{2\sqrt{0,05\ 37\ 91\ 15\ 27\ 14\ 67\ 11}} l = \\
 &= \frac{1}{2 \times 0,23\ 19\ 27\ 20} l = \frac{1}{0,46\ 38\ 58\ 40} l = \boxed{2,15\ 58\ 30\ 31... l}
 \end{aligned}$$

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3] (ver lám. 33), tendremos:

$$\begin{aligned}
 [b] &= \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{(2,15\ 58\ 30\ 31... l)^2 - \frac{l^2}{4}} = \sqrt{2,15\ 58\ 30\ 31^2 - 0,25} l = \\
 &= \sqrt{4,64\ 76\ 04\ 32\ 55\ 14\ 69\ 61 - 0,25} l = \sqrt{4,39\ 76\ 04\ 32\ 55\ 14\ 69\ 61} l = \\
 &= \boxed{2,09\ 70\ 46\ 57... l}
 \end{aligned}$$

Radio "d₃" de la circunferencia circunscrita a una cara triangular regular de lado "l"

1

2

3

1. The first part of the book is devoted to a general

discussion of the principles of the theory.

2. The second part is devoted to a detailed

examination of the various methods of

the solution of the problem.

3. The third part is devoted to a

discussion of the results of the

investigation.

4. The fourth part is devoted to a

discussion of the

conclusions.

5. The fifth part is devoted to a

discussion of the

Se demuestra en Geometría es

$$d_3 = \frac{\sqrt{3}}{3} l$$

Radio "d₅" de la circunferencia circunscrita a una cara pentagonal regular, de lado "l"

Se demuestra en Geometría es

$$d_5 = \sqrt{\frac{5 + \sqrt{5}}{10}} l$$

Radio "c₃" de la esfera tangente a las caras triangulares regulares de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} c_3 &= \sqrt{a^2 - (d_3)^2} = \sqrt{(2,15\ 58\ 30\ 31...l)^2 - \left(\frac{\sqrt{3}}{3}l\right)^2} = \\ &= \sqrt{4,64\ 76\ 04\ 32\ 55\ 14\ 69\ 61 - \frac{1}{3} \times l^2} = \\ &= \sqrt{4,31\ 42\ 70\ 99\ 21\ 81\ 36\ 28} \times l = \boxed{2,07\ 70\ 82\ 33... l} \end{aligned}$$

Radio "c₅" de la esfera tangente a las caras pentagonales regulares de lado "l"

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

The first part of the problem is to find the area of the rectangle. The area of a rectangle is found by multiplying the length by the width. In this case, the length is $\frac{1}{2}$ and the width is $\frac{3}{4}$.

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

The second part of the problem is to find the perimeter of the rectangle. The perimeter of a rectangle is found by adding the length and width together and then multiplying by 2. In this case, the length is $\frac{1}{2}$ and the width is $\frac{3}{4}$.

$$2 \times \left(\frac{1}{2} + \frac{3}{4} \right) = 2 \times \frac{5}{4} = \frac{5}{2}$$

$$\frac{5}{2} = 2 \frac{1}{2}$$

$$\frac{3}{8} = 0.375$$

The final answer is that the area of the rectangle is $\frac{3}{8}$ and the perimeter is $2 \frac{1}{2}$.

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned}
 \boxed{C_5} &= \sqrt{a^2 - (d_5)^2} = \sqrt{(2, 15 \ 58 \ 30 \ 31 \dots \ell)^2 - \left(\sqrt{\frac{5+\sqrt{5}}{10}} \ell\right)^2} = \\
 &= \sqrt{(2, 15 \ 58 \ 30 \ 31 \dots)^2 - \frac{5+\sqrt{5}}{10} \times \ell} = \\
 &= \sqrt{4, 64 \ 76 \ 04 \ 32 \ 55 \ 14 \ 69 \ 61 - 0, 72 \ 36 \ 06 \ 77 \ 77 \ 49 \ 97 \ 92 \times \ell} \\
 &= \sqrt{3, 92 \ 39 \ 97 \ 52 \ 77 \ 64 \ 71 \ 69 \times \ell} = \boxed{1, 98 \ 09 \ 08 \ 26 \dots \ell}
 \end{aligned}$$

Ángulo rectilíneo " α_3 " del diedro formado por una cara triangular, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se determina en función de su tangente, por la fórmula general [5] (ver lám. 33):

$$\begin{aligned}
 \boxed{tg \alpha_3} &= \frac{2 C_3}{\sqrt{4 (d_3)^2 - \ell^2}} = \frac{2 \times 2, 07 \ 70 \ 82 \ 33 \dots \ell}{\sqrt{4 \left(\frac{\sqrt{3}}{3} \ell\right)^2 - \ell^2}} = \frac{4, 15 \ 41 \ 64 \ 66}{\sqrt{4 \times \frac{3}{9} - 1}} = \\
 &= 4, 15 \ 41 \ 64 \ 66 \times \sqrt{3} = 7, 19 \ 52 \ 24 \ 26 \dots \ell
 \end{aligned}$$

$$\ell \text{ } tg \alpha_3 = 0, 857 \ 04 \ 44$$

$$\boxed{\alpha_3 = 82^\circ \ 5' \ 15,6''}$$

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Ángulo rectilíneo " α_5 " del diedro formado por una cara pentagonal regular, con el plano diametral del arquimedianos que pasa por una arista de aquella

Se determina en función de su tangente, por la fórmula general [6] (ver lám. 33)

$$\boxed{\frac{1}{2} \alpha_5} = \frac{2 c_5}{\sqrt{4 (d_5)^2 - l^2}} = \frac{2 \times 1,98\ 09\ 08\ 26 \cdot l}{\sqrt{4 \left(\sqrt{\frac{5+\sqrt{5}}{10}} \cdot l \right)^2 - l^2}} = \frac{3,96\ 18\ 16\ 52}{\sqrt{4 \times \frac{5+\sqrt{5}}{10} - 1}} =$$

$$= \frac{3,96\ 18\ 16\ 52}{1,37\ 63\ 81\ 92} = 2,87\ 84\ 38\ 19...$$

$$\lg \operatorname{tg} \alpha_5 = 0,45\ 91\ 554$$

$$\boxed{\alpha_5 = 70^\circ\ 50'\ 31,8''}$$

Ángulo rectilíneo " ψ_{3-3} " del diedro formado por dos caras triangulares.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\psi_{3-3}} = 2 \alpha_3 = 2 \times (82^\circ\ 5'\ 15,6'') = \boxed{164^\circ\ 10'\ 31,2''}$$

Ángulo rectilíneo " ψ_{3-5} " del diedro formado por una cara triangular y una pentagonal

Dear Sir: I have the honor to acknowledge the receipt of your letter of the 28th inst. and in reply to inform you that the same has been forwarded to the proper authorities for their consideration.

I am, Sir, very respectfully,
Your obedient servant,

Wm. H. C. [Signature]

Secretary of the Board

Chicago, Ill.

Very truly yours,
[Signature]

Enclosed for you are two copies of the report of the committee on the subject of the proposed amendment to the constitution of the American Medical Association, which was adopted at the annual meeting of the Association held at St. Louis, Mo., in May, 1913.

I am, Sir, very respectfully,
Your obedient servant,
[Signature]

Aplicando la fórmula general [4] (ver fórm. 32)

$$\boxed{\varphi_{3-5}} = \alpha_3 + \alpha_5 = 82^\circ 5' 15,6'' + 70^\circ 50' 31,8'' =$$

$$= \boxed{152^\circ 55' 47,4''}$$

Área lateral "S" del arquimedeano

Se compone de 80 caras triangulares y 12 pentagonales, regulares y de lado "l"; la superficie total será:

$$\boxed{S} = 80 \times \frac{\sqrt{3}}{4} l^2 + 12 \times \frac{\sqrt{25 + 10\sqrt{5}}}{4} l^2 = \left[20\sqrt{3} + 3\sqrt{25 + 10\sqrt{5}} \right] \times l^2 =$$

$$= \left[34,6410162 + 20,6457288 \right] \times l^2 = \boxed{55,2867450... l^2}$$

Volumen "V" del arquimedeano

Se compone de la suma de 80 pirámides de base triangular y altura "c₃" y de 12 pirámides de base pentagonal regular y altura "c₅"; su valor será:

$$\boxed{V} = \left[80 \times \frac{\sqrt{3}}{4} l^2 \times \frac{c_3}{3} + 12 \times \frac{\sqrt{25 + 10\sqrt{5}}}{4} l^2 \times \frac{c_5}{3} \right] =$$

$$= \left[34,6410162 \times \frac{2,0770823}{3} + 20,6457288 \times \frac{1,9809083}{3} \right] \times l^3 =$$

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$$= [23, 98 \ 40 \ 80 \ 5 + 13, 63 \ 24 \ 31 \ 8] \times l^3 = [37, 61 \ 65 \ 12 \ 3 \dots \times l^3]$$

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\frac{l^2}{2\sqrt{l^2 - m^2}}$	2, 15 58 30..... l
b	$\sqrt{a^2 - \frac{l^2}{4}}$	2, 09 70 47..... l
c_3	$\sqrt{a^2 - (d_3)^2}$	2, 07 70 82..... l
c_5	$\sqrt{a^2 - (d_5)^2}$	1, 98 09 08..... l
d_3	$\frac{\sqrt{3}}{3} l$	0, 57 73 50..... l
d_5	$\sqrt{\frac{5 + \sqrt{5}}{10}} l$	0, 85 06 51..... l
m	$\frac{l}{2 \operatorname{sen} \beta} l$	0, 92 27 33..... l
α_3	$\operatorname{tg} \alpha_3 = \frac{2c_3}{\sqrt{4(d_3)^2 - l^2}}$	$\operatorname{tg} \alpha_3 = 7, 19 \ 52 \ 24 \dots$ $\alpha_3 = 82^\circ \ 5' \ 15,6''$
α_5	$\operatorname{tg} \alpha_5 = \frac{2c_5}{\sqrt{4(d_5)^2 - l^2}}$	$\operatorname{tg} \alpha_5 = 2, 87 \ 84 \ 28 \dots$ $\alpha_5 = 70^\circ \ 50' \ 31,8''$
φ_{3-3}	$\alpha_2 + \alpha_3$	$\varphi_{3-3} = 164^\circ \ 10' \ 31,2''$
φ_{3-5}	$\alpha_3 + \alpha_5$	$\varphi_{3-5} = 152^\circ \ 55' \ 47,4''$
S	$[20\sqrt{3} + 3\sqrt{25 + 10\sqrt{5}}] \times l^2$	55, 28 67 45.... l^2
V	$[20\sqrt{3} \times \frac{c_3}{3} + 3\sqrt{25 + 10\sqrt{5}} \times \frac{c_5}{3}] \times l^3$	37, 61 65 12..... l^3
β	$\cos \beta = \sqrt{\frac{15+1}{32}} + \sqrt{\frac{17+27\sqrt{5}}{13824}} + \sqrt{\frac{\sqrt{5}+1}{32}} - \sqrt{\frac{17+27\sqrt{5}}{13824}}$ $\operatorname{sen} \beta = \sqrt{1 - \cos^2 \beta}$	$\cos \beta = 0, 85 \ 77 \ 80 \ 69 \dots$ $\beta = 30^\circ \ 55' \ 54,1''$ $\operatorname{sen} \beta = 0, 51 \ 40 \ 15 \ 84 \dots$

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Después del cálculo de las magnitudes principales, vamos a proceder a la representación gráfica del Arquimediano II, de lado dado, en la lámina n° 34.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotas complementarias cuyo cálculo estudiaremos posteriormente. Todas las magnitudes las obtendremos en función del lado " l_{II} " del arquimediano, de 25,5 mm (25,51 22 16)

Calculamos previamente las siguientes magnitudes:

$$l_{II} = \quad \quad \quad = 25,5 \text{ mm}$$

$$a = 2,15 \ 58 \ 30 \times 25,5 = 55,0 \text{ mm}$$

$$b = 2,09 \ 70 \ 47 \times 25,5 = 53,5 \text{ mm}$$

$$c_3 = 2,09 \ 70 \ 82 \times 25,5 = 53,0 \text{ mm}$$

$$c_5 = 1,98 \ 09 \ 08 \times 25,5 = 50,5 \text{ mm}$$

$$d_3 = 0,57 \ 53 \ 50 \times 25,5 = 14,7 \text{ mm}$$

$$d_5 = 0,85 \ 06 \ 51 \times 25,5 = 21,7 \text{ mm}$$

El orden de operaciones del trazado gráfico (lam. 34) es el siguiente:

1° Situar el centro O, de coordenadas 72, 72, 85 mm

2° Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio $a = 55 \text{ mm}$.

3° Representar en I, II y III la cara pentagonal

1-2-3-4-5, supuesto el poliedro colocado con dicha cara paralela a II y un lado (1-5) perpendicular a I. (utilizarse la cota " c_5 " en I y III).

4° Obtener en I, II y III las proyecciones del vértice 6 de la cara contigua triangular de arista 1-5 hasta colocar el vértice 6 sobre la esfera circunscrita. Para ello se hará centro en $\frac{1}{2}$, con radio igual a la altura de la cara 1-5-6 y se trazará un arco que corte en 6_I a la esfera circunscrita.

5° Determinar las proyecciones en II y III de dicho vértice 6, y seguidamente en I, II y III de los vértices 7 al 10 (los vértices 6 al 10 son los de un pentágono regular de plano paralelo al II).

6° Determinar en I la posición del vértice 14 (la arista 3-14 es paralela a I) sobre la esfera circunscrita, lo que nos permite obtener en II y III las proyecciones de dicho vértice. Los vértices 11, 12, 13 y 15 se pueden obtener seguidamente en sus tres proyecciones, puesto que el pentágono que se obtiene al unir los 11 al 15 es regular y su plano paralelo a II.

7° Para la obtención gráfica de las proyecciones de los vértices 16 al 20, 21 al 25 y 26 al 30, que nos faltan para conseguir la representación de la mitad superior de este poliedro, tendríamos que efectuar el giro de las caras contiguas alrededor de aristas ya determinadas. Esto giro presenta cierta dificultad al ser todas las aristas

No.	Name of the person	Date
1	John Doe	1914
2	Jane Smith	1915
3	Robert Brown	1916
4	Mary White	1917
5	James Black	1918
6	Elizabeth Green	1919
7	William Red	1920
8	Margaret Blue	1921
9	Charles Yellow	1922
10	Anna Pink	1923
11	George Grey	1924
12	Lillian Purple	1925
13	Frank Brown	1926
14	Alice Green	1927
15	David White	1928

del contorno poligonal ya obtenido (por 6-12-7-13-8-14-9-15-10-11-5 en planos I, II) oblicuos a I, II, lo cual nos obliga a cambios de plano para conseguir la perpendicular, de la arista tomada como eje de giro, a uno de los planos de proyección. Se simplifica este proceso si precisamente conocemos las distancias de dichos puntos al plano paralelo a II que pasa por O. Dichas distancias pueden obtenerse analíticamente, teniendo en cuenta que los vértices 16 al 20 son a su vez vértices de un pentágono regular cuyo plano es paralelo a II. Lo mismo ocurre con los 21 al 25 y los 26 al 30.

En el cálculo analítico que efectuaremos a continuación obtendremos las siguientes magnitudes (distancias de vértices al plano paralelo a II que pasa por el centro O del poliedro):

7.1 Distancia de los vértices 1 al 5 (radio " a_5 " de la esfera inscrita a una cara pentagonal).

7.2 Distancia de los vértices 6 al 10 ($g_1 - \frac{f_1}{2}$)

7.3 Distancia de los vértices 11 al 15 ($g_2 - \frac{f_2}{2}$)

7.4 Distancia de los vértices 16 al 20 ($g_3 - \frac{f_3}{2}$)

7.5 Distancia de los vértices 21 al 25 ($g_5 - \frac{f_5}{2}$)

7.6 Distancia de los vértices 26 al 30 ($g_6 - \frac{f_6}{2}$).

The first part of the report deals with the general situation of the country. It is a very interesting and informative study of the country's history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's history and its present state.

- 1. The first part of the report deals with the general situation of the country. It is a very interesting and informative study of the country's history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's history and its present state.
- 2. The second part of the report deals with the economic situation of the country. It is a very interesting and informative study of the country's economic history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's economic history and its present state.
- 3. The third part of the report deals with the social situation of the country. It is a very interesting and informative study of the country's social history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's social history and its present state.
- 4. The fourth part of the report deals with the political situation of the country. It is a very interesting and informative study of the country's political history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's political history and its present state.
- 5. The fifth part of the report deals with the cultural situation of the country. It is a very interesting and informative study of the country's cultural history and its present state. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's cultural history and its present state.

El conocimiento previo de las cotas anteriores, nos facilita el trazado gráfico para la obtención sucesiva de las proyecciones de los grupos de vértices 16 al 20, 21 al 25 y 26 al 30.

En efecto, si queremos p. e. determinar las proyecciones del vértice 18, perteneciente a la cara triangular 8-12-7, cuyos otros dos vértices 12 y 7 han sido ya obtenidos en el dibujo, tendremos que hacer girar dicha cara alrededor de la arista 12-7, hasta que el punto 18 esté sobre el plano del pentágono de vértices 16 al 20, el cual podemos situar, previamente por la cota analítica

$$g_3 - \frac{f_3}{2}$$

ya calculada.

Esta operación se efectúa fácilmente por giro sucesivo de la arista conocida del poliedro sobre dos ejes perpendiculares a II, que pasen por 12 y 7 respectivamente, que nos permite obtener, primero en II y seguidamente en I las respectivas proyecciones del mencionado vértice 18.

Obtenida ésta se pueden determinar las proyecciones de los vértices restantes del grupo 16 al 20 que forman un pentágono regular y cuyo centro en II coincide con la proyección también en II del centro del arquimediano.

Este mismo proceso se seguirá en la determinación de las proyecciones de los vértices del grupo 21

al 25 auxiliándose de la distancia analítica " $g_4 - \frac{14}{2}$ "
y finalmente para los vértices del grupo 26 al 30, me-
diante la cota " $g_5 - \frac{15}{2}$ ".

Con los pasos anteriores, hemos obtenido, en I y II,
las proyecciones de la mitad de los vértices (1 al 30)
del arquimediano pedido, correspondientes a su parte
superior: la obtención de estas proyecciones sobre III,
es inmediata, y se deduce de las de los planos I y II.

Para completar la representación del poliedro en su
otra mitad, tendremos en cuenta las siguientes pro-
piedades del arquimediano, que nos servirán para su
rápido trazado.

7.7 " Las proyecciones en II, de los vértices 31 al 60, "
" son simétricas de las de los vértices 1 al 30 "
" (ya obtenidas) con respecto a un eje que pa- "
" sa por la proyección O_{II} del centro, y que. "
" forma con el eje " $6_{II} - O_{II}$ " un ángulo ε "
" de "

7.8 " Las proyecciones en I, de los vértices 31 al 60 "
" están situadas en planos paralelos a II y a "
" distancias iguales y simétricas (ya obtenidas) de "
" también paralelos a II que pasa por la proyec- "
" ción O_I del centro.

8º Numerar los vértices.

-Como comprobación y necesaria ayuda para el trazado gráfico dado anteriormente, vamos a determinar analíticamente las siguientes magnitudes complementarias que darán mayor exactitud a dicho trazado. (ver figura de la lámina).

Altura "n" de una cara triangular

$$n = \frac{\sqrt{3}}{2} l = 0,8660254...l$$



Para el caso del dibujo, será $n = 0,8660254 \times 25,5 = 22,1 \text{ mm}$

Distancia "g₁" de los vértices 6 al 10 al plano de la cara pentagonal 1 al 5, y de los 51 al 55 a la cara pentagonal 56 al 60.

Se obtiene proyectando la altura "n" sobre el plano III; el ángulo de proyección es de

$$\varphi_{3-5} - 90^\circ = 153^\circ 55' 47,4'' - 90^\circ = 62^\circ 55' 47,4''$$

$$\begin{aligned} [g_1] &= n \times \cos 62^\circ 55' 47,4'' = \frac{\sqrt{3}}{2} \times \cos 62^\circ 55' 47,4'' \times l = \\ &= 0,3941200... \times l \end{aligned}$$

Desarrollo del cálculo anterior:

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$$\begin{aligned}
 \frac{1}{2} \lg 3 &= \frac{1}{2} \times 0,477\ 12\ 13 \text{ ---} = 0,238\ 56\ 04 \\
 + \lg. \cos\ 62^\circ\ 55'\ 47,4'' &= \text{---} = \underline{\underline{7,658\ 08\ 90}} \\
 & \quad \underline{\underline{7,896\ 64\ 97}} \\
 - \lg. 2 &= \underline{\underline{0,301\ 03\ 00}} \\
 \lg. 0,39\ 41\ 12\ 00 &= \underline{\underline{7,595\ 61\ 97}}
 \end{aligned}$$

Para el caso del dibujo, será: $g_1 = 0,39\ 41\ 12\ 00 \times 25,5 = 10,0\ \text{mm}$

Distancia "f₁" entre los dos planos paralelos a II, que contienen los vértices 6 al 10 y 51 al 55 respectivamente.

Se obtiene por diferencias de alturas "c₅" y "g₁", ya calculadas.

$$\begin{aligned}
 \boxed{f_1} &= 2(c_5 - g_1) = 2 \times (1,98\ 07\ 08\ 26 - 0,39\ 41\ 12\ 00) \times l = \\
 &= \boxed{3,17\ 35\ 92\ 52 \dots l}
 \end{aligned}$$

Para el caso del dibujo, será: $f_1 = 3,17\ 35\ 93 \times 25,5 = 80,9\ \text{mm}$

Radio "r₁" de la circunferencia circunscrita al pentágono regular de vértices 6 al 10 y 51 al 55. * (véase nota a la vuelta)

Está representado en su verdadera magnitud, en el plano II, por el segmento 6-0, suma del 6-B y del B-0.

--	--	--

1. The first part of the report is a general introduction to the subject of the study. It discusses the importance of the problem and the objectives of the research.

2. The second part of the report is a detailed description of the methods used in the study. It includes a description of the experimental design, the data collection procedures, and the statistical methods used for data analysis.

3. The third part of the report is a presentation of the results of the study. It includes a description of the data, a discussion of the findings, and a comparison of the results with previous research.

4. The fourth part of the report is a conclusion and a discussion of the implications of the study. It includes a summary of the findings, a discussion of the limitations of the study, and a discussion of the implications of the results for future research.

The results of the study show that there is a significant difference between the two groups. The first group, which received the treatment, showed a significantly higher mean score than the second group, which did not receive the treatment. This difference was statistically significant at the 0.05 level.

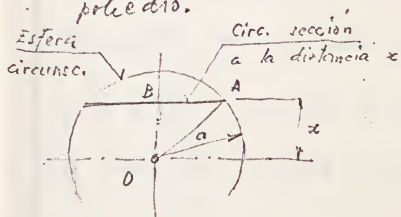
The findings of the study suggest that the treatment is effective in improving the outcome of the study. This result is consistent with previous research, which has shown that the treatment is effective in improving the outcome of the study.

The limitations of the study include the small sample size and the lack of a control group. These limitations may have affected the results of the study and should be taken into account when interpreting the findings.

The implications of the study are that the treatment is effective in improving the outcome of the study. This result is consistent with previous research, which has shown that the treatment is effective in improving the outcome of the study.

* NOTA

Este radio puede obtenerse directamente, en función de la distancia a que se encuentra el plano de la casa, del centro del poliedro (supuesta conocida), ya que en este caso dicho radio es el mismo que el de la circunferencia que se obtendría al seccionar la esfera circunscrita por el plano de la casa, puesto que los vértices 6 al 10 y 51 al 55 pertenecen a la esfera circunscrita del poliedro.



$$AB = r_1 = \sqrt{a^2 - x^2} \quad (1)$$

a = radio de la esfera circunscrita al arquimediano.

En el caso particular que nos ocupa será, haciendo

$$a = 2, 15 \ 58 \ 30 \ 31 \dots l$$

$$x = \frac{1_1}{2} = \frac{3, 17 \ 35 \ 72 \ 52}{2} l$$

$$= 1, 58 \ 67 \ 98 \ 26$$

$$r_1 = \sqrt{(2, 15 \ 58 \ 30 \ 31)^2 - (1, 58 \ 67 \ 98 \ 26)^2} \times l =$$

$$= \sqrt{4, 64 \ 76 \ 04 \ 32 \ 55 \ 14 \ 69 \ 61 - 2, 51 \ 79 \ 22 \ 37 \ 07 \ 49 \ 88 \ 76} \times l =$$

$$= \sqrt{2, 12 \ 96 \ 81 \ 95 \ 47 \ 64 \ 70 \ 25} = 1, 45 \ 93 \ 43 \ 1 \dots l$$

(Valor coincidente con el calculado anteriormente.

$$6-B = n \times \text{sen } 62^\circ 55' 47,4'' \quad (\text{ver cálculo de } g_1)$$

$$B-O = \sqrt{\frac{5+2\sqrt{5}}{20}} l \quad (\text{radio de la circunferencia inscrita al pentágono regular de una cara, de lado "l"})$$

$$\boxed{\Gamma_1} = n \times \text{sen } 62^\circ 55' 47,4'' + \sqrt{\frac{5+2\sqrt{5}}{20}} l = \left(\frac{\sqrt{3}}{2} \times \text{sen } 62^\circ 55' 47,4'' + \sqrt{\frac{5+2\sqrt{5}}{20}} l \right) =$$

$$= \boxed{1,45 \ 93 \ 43 \ 1... l}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \frac{1}{2} \lg 3 &= \frac{1}{2} \times 0,477\ 12\ 13 \quad \text{---} = 0,238\ 56\ 07 \\ + \lg \text{sen } 62^\circ 55' 47,4'' &\quad \text{---} = 7,949\ 60\ 94 \\ &\quad \quad \quad 0,188\ 17\ 01 \\ &\quad \quad \quad - \lg 2 \quad \text{---} = -0,301\ 03\ 00 \\ &\quad \quad \quad \lg \boxed{0,77\ 11\ 52\ 14} = \underline{\underline{7,887\ 14\ 01}} \end{aligned}$$

$$n \times \text{sen } 62^\circ 55' 47,4'' = 0,77\ 11\ 52\ 14$$

$$\sqrt{\frac{5+2\sqrt{5}}{20}} \quad \text{---} = 0,68\ 81\ 91,0$$

$$\boxed{\Gamma_1 = \underline{\underline{1,45\ 93\ 43\ 1... l}}}$$

Para el caso del dibujo, será: $\Gamma_1 = 1,45\ 93\ 43\ 1 \times 25,5 = 37,2\ m$

Distancia "g₂" de los vértices 11 al 15 al plano de la cara pentagonal 1 al 5, y de los 46 al 50 a la cara pentagonal 56 al 60.

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Se obtiene proyectando la arista 3-14, sobre plano III, ya que dicha arista es paralela al I.

El ángulo E de proyección se determina de la siguiente manera.

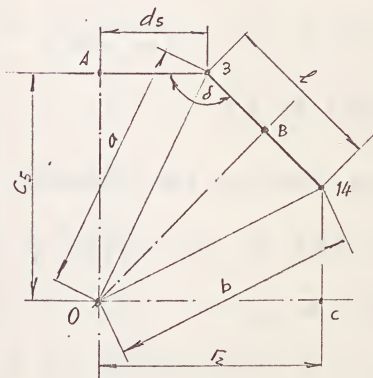


Figura 1

Consideremos (fig. 1) la sección del poliedro por el plano diametral que pasa por la arista 3-14 y el centro O del mismo. Dicho plano pasará a su vez por el centro A de la cara pentagonal 1 al 5. Unamos A, 3, B (punto medio de 3-14) y 14, con el centro O. El ángulo δ es el formado por la arista 3-14 con el plano de la cara pentagonal 1 al 5.

De la figura se deduce que

$$\cos (\widehat{A-3-O}) = \frac{ds}{a} = \frac{\sqrt{\frac{5+\sqrt{5}}{10}} l}{\frac{l^2}{2\sqrt{l^2-m^2}}} = \frac{0,8506508}{2,1558303} = 0,3945815...$$

$$\angle A-3-O = 66^\circ 45' 36,4''$$

Desarrollo del cálculo anterior:

$$\hookrightarrow 0,3945815 = 7,5961367 =$$

$$\hookrightarrow \cos (\widehat{A-3-O})$$

$$\angle A-3-O = 66^\circ 45' 36,4''$$

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations.

In the second part of the paper, we consider the case of a linear differential equation. It is shown that the problem can be reduced to a system of linear equations. The solution of this system is given in the third part of the paper.



The fourth part of the paper is devoted to a discussion of the results obtained. It is shown that the results are of great importance in the theory of differential equations.

THE END

The fifth part of the paper is devoted to a discussion of the results obtained. It is shown that the results are of great importance in the theory of differential equations.

y por otra parte

$$\cos (0-3-B) = \frac{l:2}{a} = \frac{l}{2a} = \frac{l}{2 \times \frac{l^2}{2 \sqrt{l^2 - m^2}}} = \frac{1}{2 \times 2.1558303} =$$

$$= \frac{1}{4.3116606} = 0.23192920$$

$$\angle (0-3-B) = 76^\circ 35' 21.6''$$

Desarrollo del cálculo anterior:

$$l_y \cos (0-3-B) = l_y 0.23192920 = 7.3653554$$

$$\angle (0-3-B) = 76^\circ 35' 21.6''$$

y por consiguiente

$$\angle \delta = 66^\circ 45' 36.4'' + 76^\circ 35' 21.6'' = 143^\circ 20' 58''$$

Si proyectamos la arista 2-14 sobre el plano III, el ángulo ε de proyección será:

$$\varepsilon = 143^\circ 20' 58'' - 90^\circ = 53^\circ 20' 58''$$

por lo que finalmente tendremos:

$$g_2 = l \cos 53^\circ 20' 58'' = 0.59693301 \dots l$$

Desarrollo del cálculo anterior:

$$l_y \cos 53^\circ 20' 58'' = 7.7759256$$

$$\text{Antilog } 7.7759256 = 0.59693301 \dots$$

1	2	3

Para el caso particular del dibujo, será:

$$g_2 = 0,59\ 69\ 33\ 01 \times 25,5 = 15,2\ \text{mm.}$$

Distancia " f_2 " entre los dos planos paralelos a II que contienen los vértices 11 al 15 y 46 al 50 respectivamente.

Se obtienen por diferencias de alturas " c_5 " y " g_2 ", ya calculadas.

$$\boxed{f_2} = 2 (c_5 - g_2) = 2 \times (1,98\ 09\ 08\ 26 - 0,59\ 69\ 33\ 01) \ell =$$

$$= \boxed{2,76\ 79\ 50\ 50} \ell$$

Para el caso del dibujo, será: $f_2 = 2,76\ 79\ 50\ 50 \times 25,5 = 70,6\ \text{mm.}$

Radio " r_2 " de la circunferencia circunscrita al pentágono regular de vértices 11 al 15 y 46 al 50 (ver nota al verso de la pág. 20)

$$\boxed{r_2} = \sqrt{a^2 - \left(\frac{f_2}{2}\right)^2} = \sqrt{(2,15\ 58\ 30\ 31)^2 - \left(\frac{1}{2} \times 2,76\ 79\ 50\ 50\right)^2} \times \ell$$

$$= \sqrt{(2,15\ 58\ 30\ 31)^2 - (1,38\ 39\ 75\ 25)^2} \times \ell =$$

$$= \sqrt{4,64\ 76\ 04\ 32\ 55\ 14\ 69\ 61 - 1,91\ 53\ 87\ 49\ 26\ 12\ 56\ 26} \times \ell$$

$$= \sqrt{2,73\ 22\ 16\ 83\ 29\ 02\ 13\ 36} \times \ell = \boxed{1,65\ 29\ 41\ 87} \dots \ell$$

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	By Sales	25.00
	By Interest	10.00
	By Dividend	15.00
	By Profit	20.00
	By Cash	10.00
	By Sales	15.00
	By Interest	5.00
	By Dividend	10.00
	By Profit	15.00
	By Cash	5.00
	By Sales	10.00
	By Interest	5.00
	By Dividend	10.00
	By Profit	15.00

Para el caso del dibujo será: $r_2 = 1,65\ 29\ 41\ 87 \times 25,5 = 42,1\ \text{mm.}$

Puede comprobarse el valor " r_2 " de este radio, proyectando el contorno A-3-14 (fig. 1) sobre el eje OC.

su valor se obtendrá: $\boxed{r_2} = d_5 + l (\sin \delta - 90^\circ) =$

$$= \sqrt{\frac{5 + \sqrt{5}}{10}} l + \sin (143^\circ 20' 58'' - 90^\circ) l = \left[\sqrt{\frac{5 + \sqrt{5}}{10}} + \sin 53^\circ 20' 58'' \right] \cdot l$$

$$= (0,85\ 06\ 50\ 8 + 0,80\ 22\ 91\ 1) l = \boxed{1,65\ 29\ 41\ 9...} l$$

Desarrollo del cálculo anterior:

$$l \sin 53^\circ 20' 58'' = 7,90\ 43\ 32\ 0$$

$$\text{Antilog } 7,90\ 43\ 32\ 0 = 0,80\ 22\ 91\ 1...$$

valor coincidente con el obtenido anteriormente.

Distancia " g_3 " de los vértices 16 al 20 al plano de la cara pentagonal 1 al 5, y de los vértices 41 al 45 a la cara pentagonal 56 al 60.

El cálculo de la situación de los grupos de vértices 16 al 20, 21 al 25 y 26 al 30, así como la de los grupos equidistantes del plano diametral, 31 al 35, 36 al 40 y 41 al 45. (estos g_3 , g_4 , g_5 y f_3 , f_4 , f_5), se basa en

1. The first part of the paper is devoted to a general
discussion of the problem of the existence of solutions
of the system of equations (1) and (2) under the
assumption that the functions f and g are continuous
and satisfy certain conditions. It is shown that under
these conditions the system has at least one solution
in the class of continuous functions. The proof is
based on the theorem of Poincaré-Brouwer.
2. In the second part of the paper the question of
the uniqueness of the solution is considered. It is
shown that if the functions f and g satisfy the
Lipschitz condition, then the solution is unique.
3. The third part of the paper is devoted to the
question of the stability of the solution. It is shown
that if the functions f and g satisfy certain
conditions, then the solution is stable. The proof is
based on the theorem of Liapunov.

la siguiente propiedad geométrica de este arquimediano, que enunciamos a continuación:

"El sólido que se obtiene al prolongar el plano de una cara pentagonal, hasta su intersección con las cinco caras también pentagonales que la rodean, es un dodecaedro regular", siendo el centro de las caras pentagonales del arquimediano, coincidente con el de las del dodecaedro.

De aquí se deduce que el radio C_{12} de la esfera inscrita al dodecaedro, es coincidente con el radio C_{5II} de la esfera tangente a las caras pentagonales del Arquimediano II. Así pues podemos obtener las dimensiones del dodecaedro (ver lám. 4) en función del lado "l" del arquimediano, puesto que siendo $C_{12} = C_{5II}$ se verificará que (ver fórm. 32. lám. 4):

$$\sqrt{\frac{11\sqrt{5} + 25}{40}} l_{12} = 1,98\ 09\ 08\ 26 \dots l \quad \text{de donde}$$

$$\boxed{l_{12}} = \frac{1,98\ 09\ 08\ 26}{\sqrt{\frac{11\sqrt{5} + 25}{40}}} l = \frac{1,98\ 09\ 08\ 26}{1,11\ 35\ 16\ 4} l = \boxed{1,77\ 89\ 75\ 3 \dots l}$$

Para el caso del dibujo será: $l_{12} = 1,77\ 89\ 75\ 3 \times 25,5 = 45,4\ m.m.$

Conocido el lado del dodecaedro, el del arquimediano, y el diedro de las caras del primero (ver fórm. 34, lám. 4), podemos determinar la posición del penta-

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gono del arquimédiano en relación con el de la cara del dodecaedro de centro coincidente, por proyección de las alturas " g_1 " y " g_2 " ya determinadas.

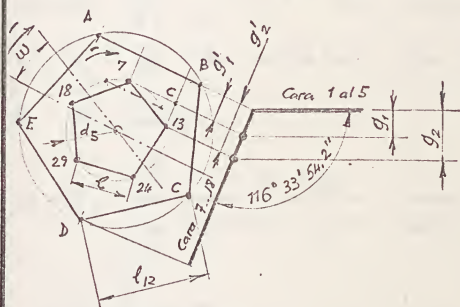


Figura 2

En la figura 2, hemos representado, a la derecha, el diedro que forman dos caras del dodecaedro, siendo el plano de la superior coincidente con el de la cara 1 al 5 del arquimédiano; la figura de la

izquierda es el abatimiento, sobre el plano del dibujo, de la cara oblicua, contigua a la anterior, que contiene, girada, una cara pentagonal del arquimédiano dado, siendo esta última una cualquiera de las cinco que rodean a la de vértices 1 al 5.

Sea A-B-C-D-E el pentágono del dodecaedro, y 7-13-24-29-18 el pentágono del arquimédiano contenido en el anterior. El ángulo de giro " w " que había que aplicar para pasar de una posición de lados paralelos en los dos pentágonos, a la posición real, se deduce del triángulo rectángulo 7-C-13 en el que

$$\angle C-7-13 = w$$

La hipotenusa 7-13 es igual a " l ", y su cateto

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined on the interval $[0, 1]$. It is shown that $f(x)$ is continuous and differentiable on this interval. The derivative of $f(x)$ is given by the formula $f'(x) = \dots$. The function $f(x)$ is also shown to be concave up on the interval $[0, 1]$.



The second part of the paper is devoted to the study of the properties of the function $g(x)$ defined on the interval $[0, 1]$. It is shown that $g(x)$ is continuous and differentiable on this interval. The derivative of $g(x)$ is given by the formula $g'(x) = \dots$. The function $g(x)$ is also shown to be concave down on the interval $[0, 1]$.

The third part of the paper is devoted to the study of the properties of the function $h(x)$ defined on the interval $[0, 1]$. It is shown that $h(x)$ is continuous and differentiable on this interval. The derivative of $h(x)$ is given by the formula $h'(x) = \dots$. The function $h(x)$ is also shown to be concave up on the interval $[0, 1]$.

C-13, es

$$C-13 = g'_2 - g'_1$$

siendo " g'_1 " y " g'_2 " las proyecciones respectivas de " g_1 " y " g_2 " de las alturas ya calculadas. Por consiguiente, tendremos

$$g'_2 - g'_1 = \frac{g_2 - g_1}{\cos(116^\circ 33' 54.2'' - 90^\circ)} = \frac{0.59 \ 69 \ 33 \ 01 - 0.39 \ 41 \ 12 \ 00}{\cos(26^\circ 33' 54.2'')} \cdot l$$

$$= \frac{0.20 \ 28 \ 31 \ 01}{0.89 \ 44 \ 27 \ 14} \cdot l = 0.22 \ 67 \ 60 \ 80 \dots l \quad \left\{ \begin{array}{l} g'_2 = 0.66 \ 73 \ 91 \ 4 \\ g'_1 = 0.44 \ 06 \ 30 \ 6 \end{array} \right.$$

y finalmente

$$\sin w = \sin(\widehat{C-7-13}) = \frac{C-13}{7-13} = \frac{g'_2 - g'_1}{l} = 0.22 \ 67 \ 60 \ 80 \dots$$

$$w = 13^\circ \ 6' \ 23.2''$$

Desarrollo del cálculo anterior:

$$\begin{array}{l} \lg \cos(26^\circ 33' 54.2'') = \bar{7}.95 \ 15 \ 45 \ 0 \\ \text{Antilog } \bar{7}.95 \ 15 \ 45 \ 0 = 0.89 \ 44 \ 27 \ 14 \end{array} \quad \left\{ \right.$$

$$\lg \sin w = \lg 0.22 \ 67 \ 60 \ 80 = \bar{7}.355 \ 56 \ 79$$

$$w = 13^\circ \ 6' \ 23.2'' \quad \checkmark$$

Date	Page	No.

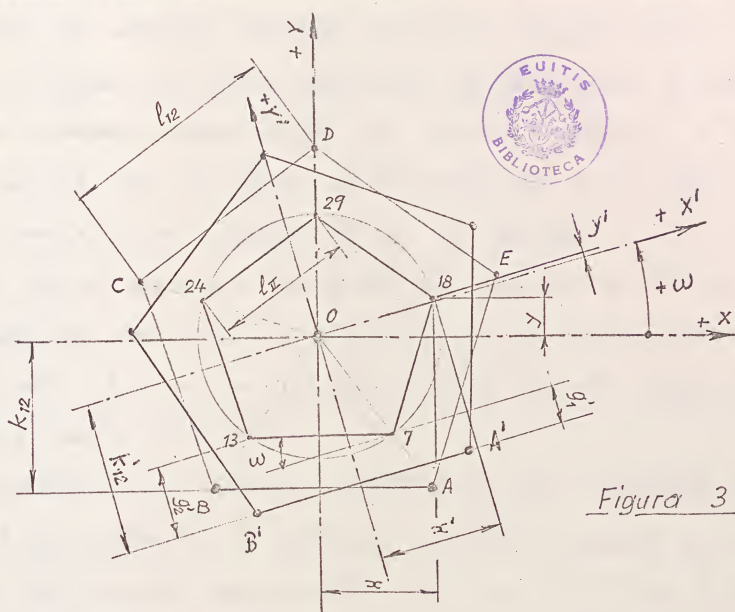


Figura 3

El proceso a seguir para determinar las constantes magnitudes analíticas " g_3 ", " g_4 " y " g_5 ", requiere obtener previamente las proyecciones de las mismas " g'_3 ", " g'_4 " y " g'_5 " sobre la cara del dodecaedro que contiene la pentagonal del Arquimediano II y ambas de centro "O" común. Los lados de ambos pentágonos regulares forman entre sí el ángulo constante " w " ya determinado, por lo cual podemos considerar que, superponiendo previamente el pentágono interior colocado con sus lados paralelos al exterior, llegaríamos a la posición final del primero mediante el giro del mismo alrededor del centro común O, de un ángulo de amplitud " w " (también se llega al mismo resultado superponiendo fijo el polígono interior y



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girando el exterior alrededor de O , el ángulo " w ").

En la figura 3 hemos planteado gráficamente el problema a resolver, conservando la misma notación de vértices que la de la figura 2. (el conjunto de la figura 3 está invertido con respecto al de la fig. 2).

Sea (fig. 3) $A-B-C-D-E$ el pentágono regular de la cara del dodecaedro de lado " l_{12} "; $7-13-24-29-18$ el del arquimedianos dado de lado " l_{II} ", en su posición inicial con sus lados paralelos y centro O común.

Consideremos un sistema cartesiano de eje X paralelo al lado $B-A$, y centro O . El problema analítico consiste en encontrar las nuevas coordenadas " x' " e " y' " de los vértices $7, 13, 24, 29, 18$ en función de las primitivas " x " e " y ", cuando los ejes, juntamente con el pentágono exterior $A-B-C-D-E$, giran alrededor del origen O , el ángulo " w ".

Las fórmulas de transformación son: *

$$\left. \begin{aligned} x' &= x \cos w + y \operatorname{sen} w \\ y' &= -x \operatorname{sen} w + y \cos w \end{aligned} \right\} \quad (1)$$

de las cuales solo necesitamos aplicar la segunda: - Conociendo " y' ", se puede calcular " g' ", por

$$g' = k'_{12} + y' \quad (2)$$

véase "Tratado de Matemáticas" por R. Doerfling; Editorial Gredos - Feli, S. A.; Barcelona 1945. - Pág. 195, párrafo 3.

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Comencemos por calcular $k_{12} = k'_{12}$ (ver fig. 3). Es la apotema de la cara pentagonal de lado " l_{12} "; su valor será pues (ver lám. 4, fórm. 39)

$$\boxed{k_{12}} = \sqrt{\frac{5+2\sqrt{5}}{20}} l_{12} = 0.6881910... l_{12} = 0.6881910... \times 1.7789753... \times l =$$

$$= \boxed{1.2242748... l} \quad \checkmark$$

Continuemos calculando las coordenadas " x " e " y " de los vértices del pentágono interior de lado " l "

$$7 \left\{ \begin{array}{l} x_7 = + \frac{1}{2} l = \text{-----} = + 0.5000000... l \\ y_7 = - \sqrt{\frac{5+2\sqrt{5}}{20}} l = \text{-----} = - 0.6881910... l \end{array} \right. \quad \checkmark$$

$$13 \left\{ \begin{array}{l} x_{13} = - \frac{1}{2} l = \text{-----} = - 0.5000000... l \\ y_{13} = - \sqrt{\frac{5+2\sqrt{5}}{20}} l = \text{-----} = - 0.6881910... l \end{array} \right. \quad \checkmark$$

$$18 \left\{ \begin{array}{l} x_{18} = + l \operatorname{sen} 54^\circ = \text{-----} = + 0.8090169... l \\ y_{18} = + d_l - l \cos 54^\circ = \sqrt{\frac{5+\sqrt{5}}{10}} l - \cos 54^\circ l = \text{-----} = + 0.2628655... l \end{array} \right. \quad \checkmark$$

$$24 \left\{ \begin{array}{l} x_{24} = - l \operatorname{sen} 54^\circ = \text{-----} = - 0.8090169... l \\ y_{24} = y_{18} = \text{-----} = + 0.2628655... l \end{array} \right. \quad \checkmark$$

$$29 \left\{ \begin{array}{l} x_{29} = \pm 0 = \text{-----} = \pm 0 \\ y_{29} = + d_l = + \sqrt{\frac{5+\sqrt{5}}{10}} l = \text{-----} = + 0.8506508... l \end{array} \right. \quad \checkmark$$

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	Dec 27	
	Dec 28	
	Dec 29	
	Dec 30	
	Dec 31	

En los cálculos anteriores intervienen los valores:

$$a) \frac{\sqrt{5 + 2\sqrt{5}}}{20} = 0,68 \ 81 \ 91 \ 0...$$

$$b) \frac{\sqrt{5 + \sqrt{5}}}{10} = 0,85 \ 06 \ 50 \ 8...$$

$$c) \cos 54^\circ; \text{ lg } \cos 54^\circ = \bar{7},76 \ 92 \ 18 \ 7 \quad \cos 54^\circ = 0,58 \ 77 \ 25 \ 3...$$

$$d) \sin 54^\circ; \text{ lg } \sin 54^\circ = \bar{7},90 \ 79 \ 57 \ 6 \quad \sin 54^\circ = 0,80 \ 90 \ 16 \ 9...$$

Para aplicar las fórmulas 1) y 2) tabulemos los cálculos respectivos a continuación:

TABLA I

Vértice	x	y	$-x \operatorname{sen} w =$ $= -0,22 \ 67 \ 60 \ 8 \ x$
7	$+0,50 \ 00 \ 00 \ 0...l$	$-0,68 \ 81 \ 91 \ 0...l$	$-0,11 \ 33 \ 80 \ 4...l$
13	$-0,50 \ 00 \ 00 \ 0...l$	$-0,68 \ 81 \ 91 \ 0...l$	$+0,11 \ 33 \ 80 \ 4...l$
24	$-0,80 \ 90 \ 16 \ 9...l$	$+0,26 \ 28 \ 65 \ 5...l$	$+0,18 \ 34 \ 53 \ 3...l$
29	± 0	$+0,85 \ 06 \ 50 \ 8...l$	$\pm 0,00 \ 00 \ 00 \ 0...l$
18	$+0,80 \ 90 \ 16 \ 9...l$	$+0,26 \ 28 \ 65 \ 5...l$	$-0,18 \ 34 \ 53 \ 3...l$

Vértice	$+y \cos w =$ $= 0,97 \ 39 \ 50 \ 2 \ y'$	$y' = -x \operatorname{sen} w +$ $+ y \cos w$	$g' = k'_2 + y' =$ $= 1,22 \ 42 \ 74 \ 8...l + y'$
7	$-0,67 \ 02 \ 63 \ 8...l$	$-0,78 \ 36 \ 44 \ 2...l'$	$+0,44 \ 06 \ 30 \ 6...l$
13	$-0,67 \ 02 \ 63 \ 8...l$	$-0,55 \ 68 \ 83 \ 4...l$	$+0,66 \ 73 \ 91 \ 4...l$
24	$+0,25 \ 60 \ 17 \ 9...l$	$+0,43 \ 94 \ 71 \ 2...l$	$+1,66 \ 37 \ 46 \ 0...l$
29	$+0,82 \ 84 \ 91 \ 5...l$	$+0,82 \ 84 \ 91 \ 5...l$	$+2,05 \ 37 \ 66 \ 3...l$
18	$+0,25 \ 60 \ 17 \ 9...l$	$+0,07 \ 25 \ 64 \ 6...l$	$+1,29 \ 68 \ 39 \ 4...l$

The following table shows the results of the experiments conducted on the 15th of May 1881. The experiments were conducted on the 15th of May 1881. The results of the experiments are as follows:

Experiment	Result	Remarks
1. The effect of the addition of a small quantity of water to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of water appears to be beneficial.
2. The effect of the addition of a small quantity of oil to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of oil appears to be beneficial.
3. The effect of the addition of a small quantity of alcohol to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of alcohol appears to be beneficial.
4. The effect of the addition of a small quantity of ether to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of ether appears to be beneficial.
5. The effect of the addition of a small quantity of benzene to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of benzene appears to be beneficial.

Experiment	Result	Remarks
6. The effect of the addition of a small quantity of carbon tetrachloride to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of carbon tetrachloride appears to be beneficial.
7. The effect of the addition of a small quantity of chloroform to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of chloroform appears to be beneficial.
8. The effect of the addition of a small quantity of carbon disulfide to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of carbon disulfide appears to be beneficial.
9. The effect of the addition of a small quantity of carbon monoxide to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of carbon monoxide appears to be beneficial.
10. The effect of the addition of a small quantity of carbon dioxide to the mixture.	The mixture became more fluid and the reaction was more rapid.	The addition of carbon dioxide appears to be beneficial.

Como puede observarse, los valores de " g_1' " y " g_2' " obtenidos en la tabla, correspondientes a los vértices 7 y 13, después del giro w , son coincidentes, como debe suceder, con los iniciales que sirvieron de base para la determinación del ángulo de giro; esto nos sirve de comprobación del cálculo realizado.

Como resultado del cálculo anterior, se han obtenido las distancias " g_1' " a " g_5' " de los vértices de una cara pentagonal del arquimedianos al lado del dodecaedro regular de caras coincidentes con las anteriores (de igual esfera circunscrita), distancias medidas sobre dicha cara. Sus valores son los siguientes:

$$\begin{aligned} g_1' &= 0, 44 \ 06 \ 30 \ 6 \dots l & (\text{vértice } 7) \\ g_2' &= 0, 66 \ 37 \ 46 \ 0 \dots l & (\quad 13) \\ g_3' &= 1, 29 \ 68 \ 39 \ 4 \dots l & (\quad 18) \\ g_4' &= 1, 66 \ 37 \ 46 \ 0 \dots l & (\quad 24) \\ g_5' &= 2, 05 \ 27 \ 66 \ 3 \dots l & (\quad 29) \end{aligned}$$

Sólo nos resta proyectar estas distancias sobre el plano III. El ángulo de proyección será (ver fig. 2)

$$116^\circ \ 33' \ 54,3'' - 90^\circ = 26^\circ \ 33' \ 54,3''$$

en el que

$$\cos 26^\circ \ 33' \ 54,3'' = 0, 89 \ 44 \ 27 \ 14 \dots$$

de donde se obtiene

$$g_1 = 0,44\ 06\ 30\ 6... \times 0,89\ 44\ 27\ 14... \ell = 0,39\ 41\ 13\ 0... \ell$$

$$g_2 = 0,66\ 73\ 91\ 4 \times \quad \quad \quad \dots \ell = 0,59\ 69\ 33\ 0... \ell$$

$$g_3 = 1,29\ 61\ 39\ 4... \times \quad \quad \quad \dots \ell = 1,15\ 99\ 28\ 4... \ell$$

$$g_4 = 1,66\ 37\ 46\ 0... \times \quad \quad \quad \dots \ell = 1,48\ 80\ 99\ 6... \ell$$

$$g_5 = 2,05\ 27\ 66\ 3... \times \quad \quad \quad \dots \ell = 1,83\ 60\ 19\ 9... \ell$$

$$g_3 = 1,15\ 99\ 28\ 4... \ell$$

Para el caso del dibujo será: $g_3 = 1,15\ 99\ 28\ 4 \times 25,5 = 29,6\ \text{mm}$

Distancia "f₃" entre los dos planos paralelos a II que contienen los vértices 16 al 20 y 41 al 45 respectivamente.

Se obtiene por diferencias de alturas "c₅" y "g₃", ya calculadas.

$$f_3 = 2 (c_5 - g_3) = 2 \times (1,98\ 09\ 08\ 3 - 1,15\ 99\ 28\ 4) \ell = 1,64\ 19\ 59\ 8... \ell$$

Para el caso del dibujo será: $f_3 = 1,64\ 19\ 59\ 8... \times 25,5 = 41,9\ \text{mm}$

Radio "r₃" de la circunferencia circunscrita al pentágono regular de vértices 16 al 20 y 41 al 45 respectivamente

(ver nota al reverso de la pág. 20)

$$r_3 = \sqrt{a^2 - \left(\frac{f_3}{2}\right)^2} = \sqrt{(2,15\ 58\ 30\ 31)^2 - \left(\frac{1}{2} \times 1,64\ 19\ 59\ 80\right)^2} \times \ell =$$

The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts.

THE STRUCTURE OF THE ATOM

The structure of the atom is determined by the laws of quantum mechanics, and the laws of quantum mechanics are in agreement with the experimental facts. The structure of the atom is determined by the laws of quantum mechanics, and the laws of quantum mechanics are in agreement with the experimental facts.

The structure of the atom is determined by the laws of quantum mechanics, and the laws of quantum mechanics are in agreement with the experimental facts. The structure of the atom is determined by the laws of quantum mechanics, and the laws of quantum mechanics are in agreement with the experimental facts.

$$= \sqrt{(2,15 \ 58 \ 30 \ 31)^2 - (0,82 \ 09 \ 79 \ 90)^2} \cdot l =$$

$$= \sqrt{4,64 \ 76 \ 04 \ 32 \ 55 \ 14 \ 69 \ 61 - 0,67 \ 40 \ 07 \ 99 \ 62 \ 04 \ 01} \cdot l =$$

$$= \sqrt{3,97 \ 35 \ 96 \ 32 \ 93 \ 10 \ 68 \ 61} \cdot l = \boxed{1,99 \ 33 \ 88 \ 2... l}$$

Para el caso del dibujo será: $r_3 = 1,99 \ 33 \ 88 \ 2... \times 25,5 = 50,8 \text{ mm}$

Distancia "g₄" de los vértices 21 al 25 al plano de la cara pentagonal 1 al 5, y de los vértices 36 al 40 a la cara pentagonal 56 al 60. (ver final cálculo "g₃")

$$\boxed{g_4 = 1,48 \ 80 \ 99 \ 6... l}$$

Para el caso del dibujo, será: $g_4 = 1,48 \ 80 \ 99 \ 6... \times 25,5 = 37,9 \text{ mm}$

Distancia "f₄" entre los planos paralelos a II que contienen los vértices 21 al 25 y 36 al 40, respectivamente.

Se obtiene por diferencias de alturas "c₅" y "g₄", ya calculadas.

$$\boxed{f_4} = 2 (c_5 - g_4) = 2 (1,98 \ 07 \ 08 \ 3 - 1,48 \ 80 \ 99 \ 6) l = \boxed{0,98 \ 56 \ 17 \ 4... l}$$

Para el caso del dibujo, será: $f_4 = 0,98 \ 56 \ 17 \ 4... \times 25,5 = 25,1 \text{ mm}$

1. The first part of the paper is devoted to a general
 introduction of the subject and a brief review of the
 literature. The second part is devoted to a detailed
 description of the experimental apparatus and the
 results of the measurements. The third part is devoted to a
 discussion of the results and a comparison with the
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 theoretical predictions. The fourth part is devoted to a
 conclusion and a summary of the results.

Radio "r₄" de la circunferencia circunscrita al pentágono regular de vértices 21 al 25 y 36 al 40 respectivamente.

(ver nota al reverso de la página 20)

$$\boxed{r_4} = \sqrt{a^2 - \left(\frac{f_4}{2}\right)^2} = \sqrt{(2,15\ 58\ 30\ 31)^2 - \left(\frac{1}{2} \times 0,98\ 56\ 17\ 40\right)^2} \times l =$$

$$= \sqrt{(2,15\ 58\ 30\ 31)^2 - (0,49\ 28\ 08\ 70)^2} \times l =$$

$$= \sqrt{4,64\ 76\ 04\ 32\ 55\ 14\ 69\ 61 - 0,24\ 28\ 60\ 41\ 47\ 95\ 69\ 00} \times l =$$

$$= \sqrt{4,40\ 47\ 43\ 91\ 07\ 19\ 00\ 61} \times l = \boxed{2,09\ 87\ 48\ 2... l}$$

Para el caso del dibujo, será: $r_4 = 2,09\ 87\ 48\ 2... \times 25,5 = 53,5\ \text{mm}$

Distancia "g₅" de los vértices 26 al 30 al plano de la cara pentagonal 1 al 5, y de los vértices 31 al 35 a la cara pentagonal 56 al 60. (ver final cálculo "g₃")

$$\boxed{g_5 = 1,83\ 60\ 49\ 9... l}$$

Para el caso del dibujo, será: $g_5 = 1,83\ 60\ 49\ 9... \times 25,5 = 46,8\ \text{mm}$

Distancia "f₅" entre los dos planos paralelos a II que contienen los vértices 26 al 30 y 31 al 35, respectivamente.

Subject: Mathematics
Chapter: Algebra
Topic: Linear Equations in Two Variables

Q.1. Solve the following system of linear equations by the substitution method:
$$x + y = 5$$
$$2x - y = 1$$

Solution: From equation (1), we have $y = 5 - x$. Substituting this value of y in equation (2), we get

$$2x - (5 - x) = 1$$
$$2x - 5 + x = 1$$
$$3x - 5 = 1$$
$$3x = 1 + 5$$
$$3x = 6$$
$$x = \frac{6}{3}$$
$$x = 2$$

Substituting $x = 2$ in equation (1), we have
 $2 + y = 5$
$$y = 5 - 2$$
$$y = 3$$

$$\therefore x = 2, y = 3$$

Q.2. A two-digit number is such that the sum of its digits is 9. If the digits are reversed, the number obtained is 27 less than the original number. Find the number.

Solution: Let the tens digit be x and the units digit be y . Then the number is $10x + y$.
According to the question, $x + y = 9$... (1)
If the digits are reversed, the number is $10y + x$.
According to the question, $10y + x = 10x + y - 27$... (2)

Se obtiene por diferencia de alturas " c_5 " y " g_5 ", ya calculadas.

$$f_5 = 2 (c_5 - g_5) = 2 \times (1.9809083 - 1.8360499) = 0.2897168... l$$

Para el caso del dibujo, será: $f_5 = 0.2897168... \times 25.5 = 7.4 \text{ mm}$

Radio " r_5 " de la circunferencia circunscrita al pentágono regular de vértices 26 al 30 y 31 al 35 respectivamente.

(ver nota al reverso de la pág. 20)

$$r_5 = \sqrt{a^2 - \left(\frac{f_5}{2}\right)^2} = \sqrt{(2.15583031)^2 - \left(\frac{1}{2} \times 0.28971680\right)^2} \times l =$$

$$= \sqrt{(2.15583031)^2 - (0.14485840)^2} \times l =$$

$$= \sqrt{4.6476043255146961 - 0.0209839560505600} \times l =$$

$$= \sqrt{4.6266203694641361} \times l = 2.1509580... l$$

Para el caso del dibujo, será: $r_5 = 2.1509580... \times 25.5 = 54.9 \text{ mm}$

Ángulo " ϵ " que forma el eje de simetría " h " en la proyección II, con el eje paralelo a X.

Refiriéndonos a la proyección II (lámin. 34), el triángulo-

...

...

...

...

...

...

...

...

...

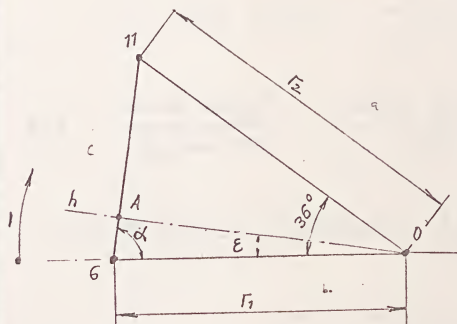


Figura 4

11-0-6 (fig. 4), tiene los valores ya calculados $6-0 = r_1$, $11-0 = r_2$ y el ángulo que forman estos lados es de $\frac{360}{5} : 2 = 36^\circ$,

La altura OA de este triángulo, correspondiente a 0, es el eje "h" de

simetría de las proyecciones en II de los vértices del arquimedianos; este eje forma un ángulo "E" (negativo) con el lado 0-6.

En la resolución trigonométrica de triángulos oblicuángulos, cuando se conocen dos lados y el ángulo comprendido, se obtiene otro de sus ángulos por la fórmula

$$\operatorname{tg} \alpha = \frac{a \operatorname{sen} \gamma}{b - a \cos \gamma}$$

que aplicada al caso particular de la figura 4, haciendo

$$a = r_2; \quad b = r_1 \quad \gamma = 36^\circ \quad \text{y} \quad \alpha = \widehat{0-6-11}$$

y siendo

$$\operatorname{sen} \gamma = \operatorname{sen} 36^\circ = 0.5877853 \dots$$

$$\cos \gamma = \cos 36^\circ = 0.8090169 \dots$$

y que

$$\operatorname{tg} \alpha = \operatorname{ctg} \epsilon, \quad \text{tendremos:}$$

The first part of the
 report is devoted to a
 description of the
 work done during the
 year. It is divided into
 two main sections, the
 first of which deals with
 the work done in the
 laboratory, and the second
 with the work done in the
 field.



The second part of the report
 is devoted to a discussion of
 the results of the work. It
 is divided into two main
 sections, the first of which
 deals with the results of the
 laboratory work, and the
 second with the results of the
 field work. The results of the
 laboratory work are discussed
 in terms of the work done
 during the year, and the
 results of the field work are
 discussed in terms of the work
 done during the year.

The third part of the report
 is devoted to a summary of
 the work done during the
 year. It is divided into two
 main sections, the first of
 which deals with the work
 done in the laboratory, and
 the second with the work
 done in the field. The
 summary of the work done
 in the laboratory is given
 in terms of the work done
 during the year, and the
 summary of the work done
 in the field is given in terms
 of the work done during the
 year.

$$\frac{1}{\theta} \alpha = \operatorname{ctg} \varepsilon = \frac{r_2 \operatorname{sen} 36^\circ}{r_1 - r_2 \operatorname{ctg} 36^\circ} \quad \text{de donde}$$

$$\frac{1}{\theta} \varepsilon = \frac{r_1 - r_2 \operatorname{ctg} 36^\circ}{r_2 \operatorname{sen} 36^\circ} = \frac{4,45 \ 93 \ 43 \ 1... \ell - 1,65 \ 29 \ 41 \ 9 \times 0,80 \ 90 \ 16 \ 7.. \ell}{4,65 \ 29 \ 41 \ 9 \times 0,58 \ 77 \ 85 \ 3.. \times \ell} =$$

$$= \frac{4,45 \ 93 \ 43 \ 1 - 1,33 \ 72 \ 57 \ 9}{0,97 \ 15 \ 75 \ 0} = \frac{0,12 \ 20 \ 85 \ 2}{0,97 \ 15 \ 75 \ 0} = 0,12 \ 56 \ 57 \ 0$$

$$\varepsilon = 7^\circ \ 9' \ 43,5''$$

$$\hookrightarrow \frac{1}{\theta} \varepsilon = 7,09 \ 91 \ 86 \ 7 = \hookrightarrow 0,12 \ 56 \ 57 \ 0$$

En la página siguiente damos un resumen tabulado de las magnitudes complementarias calculadas.

FIGURA CORPÓREA

Se obtiene por acoplamiento de 80 triángulos equiláteros y 12 pentágonos regulares, de lado 25,5 mm de forma que en cada vértice concurren 4 triángulos y 1 cuadrado.

CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
n	$\frac{\sqrt{3}}{2} \ell$	0.86 60 25... ℓ
f_1	$2 (c_5 - g_1)$	3.17 35 93... ℓ
f_2	$2 (c_5 - g_2)$	2.76 79 51... ℓ
f_3	$2 (c_5 - g_3)$	1.64 19 60... ℓ
f_4	$2 (c_5 - g_4)$	0.98 56 17... ℓ
f_5	$2 (c_5 - g_5)$	0.28 97 17... ℓ
g_1	$\frac{\sqrt{3}}{2} \times \cos 62^\circ 55' 47.4'' \ell$	0.39 41 12... ℓ
g_2	$\cos 53^\circ 20' 58'' \ell$	0.59 69 33... ℓ
g_3	$g'_3 \cos 26^\circ 33' 54.2''$	1.15 79 28... ℓ
g_4	$g'_4 \cos 26^\circ 33' 54.2''$	1.48 81.00... ℓ
g_5	$g'_5 \cos 26^\circ 33' 54.2''$	1.83 60 50... ℓ
r_1	$\sqrt{a^2 - \left(\frac{f_1}{2}\right)^2}$	1.45 93 43... ℓ
r_2	$\sqrt{a^2 - \left(\frac{f_2}{2}\right)^2}$	1.65 29 42... ℓ
r_3	$\sqrt{a^2 - \left(\frac{f_3}{2}\right)^2}$	1.99 33 88... ℓ
r_4	$\sqrt{a^2 - \left(\frac{f_4}{2}\right)^2}$	2.09 87 48... ℓ
r_5	$\sqrt{a^2 - \left(\frac{f_5}{2}\right)^2}$	2.15 09 58... ℓ
w	$\sin w = \frac{g_2 - g_1}{\cos 26^\circ 33' 54.2''}$	$\sin w = 0.22 67 60.8$ $w = 13^\circ 6' 23.2''$
ϵ	$\tan \epsilon = \frac{r_1 - r_2 \cos 36^\circ}{r_2 \sin 36^\circ}$	$\tan \epsilon = 0.12 56 57.0$ $\epsilon = 7^\circ 9' 43.5''$

ESTUDIO COMPLEMENTARIO AL CÁLCULO DEL
RADIO "m" DE LA CIRCUNFERENCIA CIRCUN-
SCRITA AL POLÍGONO OBTENIDO AL UNIR LOS EX-
TREMOS DE LAS ARISTAS DE UN ÁNGULO SÓ-
LIDO DE UN POLIEDRO ARQUIMEDIANO. - - -

En el estudio de dicha magnitud "m" en los arquimedianos I y II, y exclusivamente en éstos, se presenta el problema geométrico y analítico de la determinación de dicha magnitud, en el que el polígono que se forma al unir los extremos de las aristas de un ángulo sólido, es un pentágono irregular (plano), de cuatro lados iguales y un quinto desigual, mayor que los anteriores.

Enfocado desde el punto de vista geométrico, el problema planteado puede enunciarse de la forma siguiente:

PROBLEMA

" Inscribir un pentágono irregular "
" en una circunferencia, tienien - "
" do aquel cuatro lados iguales "
" (l), y uno desigual (a), siendo "
" $a > l$. _____ "

Para que el problema sea posible, es preciso demostrar:

- 1.º Si existe algún límite entre la relación de "a" y "l", en el que el problema no tenga solución
- 2.º Si existe, al menos, una cierta relación entre "a" y "l", en que se pueda construir un pentágono de las condiciones del enunciado, y que esté a su vez "inscrito en una circunferencia"

En efecto, y con respecto al punto 1.º, construyamos (fig. 1) un pentágono irregular A-B-C-D-E, con las condiciones del enunciado,

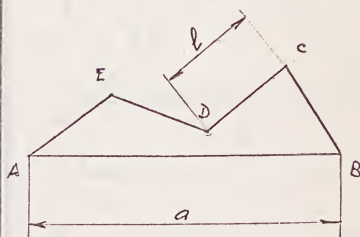


Figura 1

es decir, en el que se verifique que $BC = CD = DE = EA = l$

y $AB = a$; siendo $a > l$, el cual podrá obtenerse siempre y cuando se tenga que

$$BC + CD + DE + EA > AB,$$

o sea que $4l > a$, o en equivalente

$$\frac{a}{l} < 4 \quad (1)$$

para valores de $\frac{a}{l} \geq 4$, el problema no tiene solución geométrica. Por el contrario, para valores de

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations. The second part is devoted to the construction of the solution. It is shown that the solution can be obtained by the method of variation of parameters. The third part is devoted to the study of the properties of the solution. It is shown that the solution is unique and that it satisfies the boundary conditions.

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The seventh part is devoted to the study of the properties of the solution. It is shown that the solution is unique and that it satisfies the boundary conditions. The eighth part is devoted to the study of the properties of the solution. It is shown that the solution is unique and that it satisfies the boundary conditions. The ninth part is devoted to the study of the properties of the solution. It is shown that the solution is unique and that it satisfies the boundary conditions.

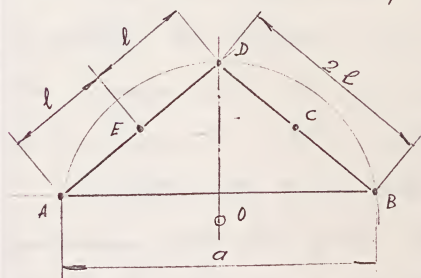
$0 < \frac{a}{l} < 4$, el problema tiene siempre solución.

Esto nos indica que "l" puede variar entre los valores

$$\frac{a}{4} < l < \infty \quad (2)$$

Vamos a demostrar seguidamente la propiedad de que un pentágono irregular de las condiciones del enunciado, en el que se verifique la condición (2), es inscriptible en una circunferencia (punto 2º).

Para ello supongamos el pentágono dado como un sistema de varillas rígidas, articuladas en sus vértices;



de esta forma podemos transformar el pentágono de la figura 1 en un triángulo isósceles, según se representa en la figura 2,

en el que los vértices E y C

estén alineados con los A, D y B, D, respectivamente.

Tracemos a continuación la circunferencia de centro O, circunscrita al triángulo A-D-B. Los puntos E y C son pues, interiores a dicha circunferencia, por ser puntos intermedios de los segmentos A-D y D-B, y éstos cuerdas de dicha circunferencia.

Transformemos de nuevo el pentágono de la figura 1, hasta conseguir que los vértices E, D y C,

22-12-72

The first part of the paper is devoted to a general discussion of the problem of the existence of a solution of the differential equation

$$y'' + p(x)y' + q(x)y = r(x)$$

under the assumption that $p(x)$, $q(x)$ and $r(x)$ are continuous functions of x in the interval (a, b) . It is shown that if $p(x)$ and $q(x)$ are bounded in this interval, then there exists a unique solution of the equation which satisfies the initial conditions

$y(a) = y_0$, $y'(a) = y'_0$.



It is also shown that if $p(x)$ and $q(x)$ are not bounded in the interval (a, b) , then there may or may not exist a solution of the equation which satisfies the initial conditions. In this case, the existence of a solution depends on the particular form of the functions $p(x)$ and $q(x)$.

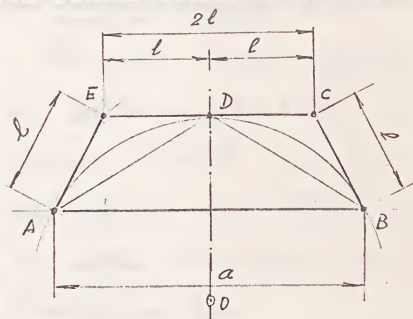


Figura 3

estén alineado, según se representa en la figura 3; el pentágono dado se transforma en el trapecio isósceles $A-B-C-E$, en el que el vértice D es el centro de la base $E-C$.

Unamos a continuación D , con A y B , y tracemos la circunferencia circunscrita al triángulo $A-D-B$ (también isósceles); la recta $E-C$, será pues tangente a dicha circunferencia (su centro O está en el eje de simetría del trapecio $A-B-C-E$ y del triángulo $A-D-B$). Por consiguiente los puntos E y C , pertenecientes a dicha tangente serán exteriores a dicha circunferencia.

Considerando que podemos pasar de una forma continua del triángulo de la figura 2 al trapecio de la figura 3 mediante el acercamiento del vértice D a su lado $A-B$, supuesto éste inmóvil, y que en cada posición trazamos la circunferencia circunscrita al triángulo $A-D-B$, los puntos C y E , pasarán de una determinada posición interior de una circunferencia, a otra exterior momentáneamente próxima, y por consiguiente por otra intermedia que los contenga.

En esta posición límite, el pentágono dado, estará inscrito en una circunferencia (s. s. q. d.).

The first part of the problem is to find the area of the rectangle. The length is 10 units and the width is 5 units. The area is calculated by multiplying the length by the width.



The second part of the problem is to find the perimeter of the rectangle. The perimeter is the sum of all four sides. Since the length is 10 units and the width is 5 units, the perimeter is calculated by adding the lengths of all four sides.

The final part of the problem is to find the area of the triangle. The base of the triangle is the same as the length of the rectangle, which is 10 units. The height of the triangle is the same as the width of the rectangle, which is 5 units. The area is calculated by multiplying the base by the height and dividing by 2.

SOLUCION GRAFICA

No existe solución gráfica exacta, con la regla y compás, al problema planteado de encontrar la anterior circunferencia límite, ya que como veremos a continuación, la solución analítica para la obtención del radio "m" de dicha circunferencia conduce a la resolución de una ecuación cúbica.

SOLUCION ANALITICA

Supongamos el problema resuelto en un caso cualquiera,

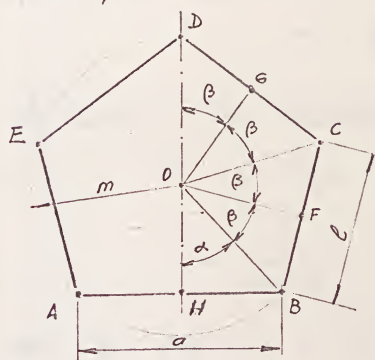


Figura 4

en que se cumpla la condición (2) de ser $\frac{a}{4} < l < \infty$ según se representa en la figura 4.*

La solución de este problema se consigue al obtener la magnitud del radio "m" de la circun-

ferencia circunscrita al pentágono irregular A-B-C-D-E que tiene 4 lados iguales (l) y uno desigual (a) ($a > l$).

* la figura 4 es análoga a la fig. 1 de la lámina 33, que utilizamos en la solución del Arquimedianos I

The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the structure of the atom is determined by the laws of quantum mechanics.

The second part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the structure of the atom is determined by the laws of quantum mechanics.



The third part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the structure of the atom is determined by the laws of quantum mechanics.

Dicho pentágono tendrá un eje de simetría DH sobre el que se encuentra el centro O de la circunferencia pedida; dicho eje es la mediatriz del lado AB (a), y pasará pues por el punto H, medio del segmento AB.

Unamos los vértices B y C con el centro O, así como los F y G, centro de los lados DC y CD respectivamente. Con esto se nos formará el ángulo $\widehat{H-O-B} = \alpha$ y los todos iguales $\widehat{B-O-F} = \widehat{F-O-C} = \widehat{C-O-G} = \widehat{G-O-D} = \beta$, verificándose que

$$\alpha + 4\beta = \pi \quad (3)$$

El radio "m" buscado se puede obtener en función de "β" y "l", ya que $OB = \frac{BF}{\sin \beta}$, o sea

$$m = \frac{\frac{l}{2}}{\sin \beta} = \frac{l}{2 \sin \beta} \quad (4)$$

Para la obtención de "β" seguiremos el siguiente proceso, de acuerdo con la fig. 4

$$\sin \alpha = \frac{HB}{OB} = \frac{\frac{a}{2}}{m} = \frac{a}{2m} \quad (5)$$

y también

$$\sin \beta = \frac{BF}{OB} = \frac{\frac{l}{2}}{m} = \frac{l}{2m} \quad (6)$$

dividiendo (5) por (6)

$$\frac{\sin \alpha}{\sin \beta} = \frac{a}{2m} : \frac{l}{2m} = \frac{a}{l} \quad (7)$$

The first part of the work was done in the morning. The second part was done in the afternoon. The third part was done in the evening. The fourth part was done in the night. The fifth part was done in the day. The sixth part was done in the week. The seventh part was done in the month. The eighth part was done in the year. The ninth part was done in the century. The tenth part was done in the millennium.

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pero siendo " α " suplementario de " 4β ", se verificará:

$$\operatorname{sen} \alpha = \operatorname{sen} 4\beta \quad (8)$$

y teniendo en cuenta (7) y (8)

$$\frac{\operatorname{sen} \alpha}{\operatorname{sen} \beta} = \frac{\operatorname{sen} 4\beta}{\operatorname{sen} \beta} = \frac{a}{l} \quad (9)$$

Desarrollando la (9)

$$\frac{a}{l} = \frac{\operatorname{sen} 4\beta}{\operatorname{sen} \beta} = \frac{\operatorname{sen} [2 \times (2\beta)]}{\operatorname{sen} \beta} = \frac{2 \operatorname{sen} 2\beta \cos 2\beta}{\operatorname{sen} \beta} =$$

$$= \frac{2 \times 2 \operatorname{sen} \beta \cos \beta \times (2 \cos^2 \beta - 1)}{\operatorname{sen} \beta} = 4 \cos \beta (2 \cos^2 \beta - 1) =$$

$$= 8 \cos^3 \beta - 4 \cos \beta \quad \text{o sea}$$

$$8 \cos^3 \beta - 4 \cos \beta = \frac{a}{l}$$

de donde

$$\cos^3 \beta - \frac{1}{2} \cos \beta - \frac{a}{8l} = 0 \quad (10)$$

ecuación cúbica en " $\cos \beta$ ". Haciendo $\cos \beta = x$, será

$$x^3 - \frac{1}{2} x - \frac{a}{8l} = 0 \quad (11)$$

Las tres raíces de esta ecuación pueden obtenerse aplicando la "Fórmula de Cardano", que exponemos a continuación.

	<p>For the purpose of this study, the following data was collected:</p>	
	<p>The following table shows the results of the experiment:</p>	
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Para ello, transformamos previamente la ecuación (11)*, haciendo

$$p = -\frac{1}{2} \quad y \quad q = -\frac{a}{8l}$$

por lo que tendremos

$$x^3 + px + q = 0 \quad (12)$$

forma reducida de una ecuación cúbica completa.

Haciendo por otra parte

$$R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

la fórmula de Cardano expresa que

$$x_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{R}} + \sqrt[3]{-\frac{q}{2} - \sqrt{R}} \quad (13)$$

siendo x_1 una de las tres raíces de la ecuación (12). Las dos raíces restantes " x_2 " y " x_3 " se obtienen reduciendo la cúbica (12) a una cuadrada, al dividir dicha ecuación por " $x - x_1$ ". Estas dos raíces restantes pueden ser reales o imaginarias.

Para que la primera raíz " x_1 " sea real, es necesario que se verifique en (13) que

$$R \geq 0 \quad (14)$$

* Ver "Matemáticas para Ingenieros y Técnicos" de R. Dörfeling, pag. 58.- Editorial Gustavo Gili, S.A.- Barcelona 1945.

No.	Date	Page
1	Jan 1 1900	1
2	Jan 2 1900	2
3	Jan 3 1900	3
4	Jan 4 1900	4
5	Jan 5 1900	5
6	Jan 6 1900	6

por lo que también será $\sqrt{R} \neq 0$.

En el caso de ser $R > 0$, tendremos una raíz real y dos imaginarias.

Si fuese $R = 0$, las tres raíces son reales, siendo dos de ellas iguales ($x_2 = x_3$).

En el caso de ser $R < 0$ (caso irreducible), en solución obliga a operar en el campo complejo, obteniéndose tres raíces reales diferentes.

SOLUCION DEL PROBLEMA EN CASOS PARTICULARES

1º Caso Arquimédiano I

Vamos a hacer uso de la solución general planteada, a diversos casos particulares, comenzando por el del "Arquimédiano I".

En éste se verifica que $\frac{a}{l} = \sqrt{2}$, siendo por consiguiente:

$$\boxed{l} = \frac{\sqrt{2}}{2} a = \boxed{0.7071068... a}$$

en la ecuación cúbica (11) será

$$\frac{a}{8l} = \frac{\sqrt{2}}{8}$$

por lo que aquélla se transformará en

Blank body area with faint horizontal lines.

$$x^3 - \frac{1}{2}x - \frac{\sqrt{2}}{8}$$

y haciendo $p = -\frac{1}{2}$ y $q = -\frac{\sqrt{2}}{8}$

para $R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 = \left(-\frac{\sqrt{2}}{8}; 2\right)^2 + \left(-\frac{1}{2}; 3\right)^3 = \frac{1}{2^7} - \frac{1}{2^3 \times 3^3} =$

$$= \frac{2^3 \times 3^3 - 2^7}{2^{10} \times 3^3} = \frac{3^3 - 2^4}{2^7 \times 3^3} > 0 \quad \text{y por consiguiente}$$

$$\sqrt{R} = \sqrt{\frac{3^3 - 2^4}{2^7 \times 3^3}} = \sqrt{\frac{11}{3456}} > 0 \quad \text{por lo que tendremos}$$

$$x_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{R}} + \sqrt[3]{-\frac{q}{2} - \sqrt{R}} = \sqrt[3]{-\left(-\frac{\sqrt{2}}{8}; 2\right) + \sqrt{\frac{11}{3456}}} +$$

$$+ \sqrt[3]{-\left(-\frac{\sqrt{2}}{8}; 2\right) - \sqrt{\frac{11}{3456}}} = \sqrt[3]{\frac{\sqrt{2}}{16} + \sqrt{\frac{11}{3456}}} + \sqrt[3]{\frac{\sqrt{2}}{16} - \sqrt{\frac{11}{3456}}} \approx$$

$$\approx 0,84250920 = \cos \beta$$

$$\boxed{\beta = 32^\circ 35' 38,3''}$$

(ver desarrollo y aplicación de este cálculo, en lám. 33).
Las dos raíces restantes son imaginarias

2° Caso Arquimedianos II

En este se verifica que $\frac{a}{l} = \frac{\sqrt{5}+1}{2}$

siendo por consiguiente

$$\boxed{L} \cdot \frac{2}{\sqrt{5}+1} a = \frac{2(\sqrt{5}-1)}{4} a \approx \boxed{0,61803399... a}$$

Q. 1. (a)

Find the value of $\sin^{-1}(\sin \frac{\pi}{6})$

Sol. We know that $\sin^{-1}(\sin x) = x$ if x lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Here $\frac{\pi}{6}$ lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\therefore \sin^{-1}(\sin \frac{\pi}{6}) = \frac{\pi}{6}$

(b) Find the value of $\cos^{-1}(\cos \frac{5\pi}{6})$

Sol. We know that $\cos^{-1}(\cos x) = x$ if x lies in $[0, \pi]$

Here $\frac{5\pi}{6}$ lies in $[0, \pi]$

$$\therefore \cos^{-1}(\cos \frac{5\pi}{6}) = \frac{5\pi}{6}$$

(c) Find the value of $\tan^{-1}(\tan \frac{7\pi}{6})$

Sol. We know that $\tan^{-1}(\tan x) = x$ if x lies in $(-\frac{\pi}{2}, \frac{\pi}{2})$

Here $\frac{7\pi}{6}$ does not lie in $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\therefore \tan^{-1}(\tan \frac{7\pi}{6}) \neq \frac{7\pi}{6}$

$$\tan^{-1}(\tan \frac{7\pi}{6}) = \tan^{-1}(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$$

en la ecuación cúbica (11) será:

$$\frac{a}{8l} = \frac{\sqrt{5}+1}{2 \times 8} = \frac{\sqrt{5}+1}{16}$$

por lo que aquella se transformará en

$$x^3 - \frac{1}{2}x - \frac{\sqrt{5}+1}{16} = 0$$

y haciendo

$$p = -\frac{1}{2}$$

$$q = -\frac{\sqrt{5}+1}{16}$$

$$\text{será } R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 = \left(-\frac{\sqrt{5}+1}{16} : 2\right)^2 + \left(-\frac{1}{2} : 3\right)^3 = \frac{(\sqrt{5}+1)^2}{32^2} - \frac{1}{2^3 \times 3^3} =$$

$$= \frac{3 + \sqrt{5}}{16 \times 32} - \frac{1}{2^3 \times 3^3} = \frac{(3 + \sqrt{5}) \times 2^3 \times 3^3 - 2^9}{2^{12} \times 3^3} = \frac{(3 + \sqrt{5}) \times 3^3 - 2^6}{2^9 \times 3^3} = \frac{3^4 - 2^6 + 3^3 \sqrt{5}}{2^9 \times 3^3} =$$

$$= \frac{17 + 27\sqrt{5}}{13824} > 0 \quad \text{y por consiguiente} \quad \sqrt{R} > 0$$

por lo que tendremos:

$$x_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{R}} + \sqrt[3]{-\frac{q}{2} - \sqrt{R}} = \sqrt[3]{-\left(-\frac{\sqrt{5}+1}{16} : 2\right) + \sqrt{\frac{17 + 27\sqrt{5}}{13824}}} +$$

$$\sqrt[3]{-\left(-\frac{\sqrt{5}+1}{16} : 2\right) - \sqrt{\frac{17 + 27\sqrt{5}}{13824}}} = \sqrt[3]{\frac{\sqrt{5}+1}{32}} + \sqrt{\frac{17 + 27\sqrt{5}}{13824}} + \sqrt[3]{\frac{\sqrt{5}+1}{32}} + \sqrt{\frac{17 + 27\sqrt{5}}{13824}} \approx$$

$$\approx 0,85778067... = \cos(\beta)$$

$$\boxed{\beta = 30^\circ 55' 54,1''}$$

Las dos raíces restantes son imaginarias

3º caso límite en que $\frac{a}{l} = 4$

En este caso será

$$\boxed{l} = \frac{a}{4} = \boxed{0,25 \times a}$$

Blank main body area with faint horizontal lines.

en la ecuación cúbica (11) será:

$$\frac{a}{8l} = \frac{4}{8} = \frac{1}{2}$$

por lo que aquélla se transformará en

$$x^3 - \frac{1}{2}x - \frac{1}{2} = 0$$

donde se ve de inmediato que tiene una raíz real $x_1 = 1$, que la verifica, y que al dividir la ecuación por $x - x_1 = x - 1$, da lugar a otra de segundo grado con sus dos raíces restantes x_2 y x_3 imaginarias conjugadas.

Si independientemente de esto, aplicamos la fórmula general de Cardano, haciendo

$$p = -\frac{1}{2} \quad q = -\frac{1}{2}$$

$$\text{será} \quad R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 = \left(-\frac{1}{2} : 2\right)^2 + \left(-\frac{1}{2} : 3\right)^3 = \frac{1}{2^4} - \frac{1}{2^3 \times 3^3} =$$

$$= \frac{2^3 \times 3^3 - 2^4}{2^7 \times 3^3} = \frac{3^3 - 2}{2^4 \times 3^3} = \frac{25}{2^4 \times 3^3} = \frac{5^2}{2^4 \times 3^3} > 0 \quad \text{y por consiguiente}$$

$$\sqrt{R} = \sqrt{\frac{5^2}{2^4 \times 3^3}} = \frac{5}{2^2 \times 3 \times \sqrt{3}} = \frac{5\sqrt{3}}{36} > 0$$

por lo que tendremos:

$$\boxed{x_1} = \sqrt[3]{-\frac{q}{2} + \sqrt{R}} + \sqrt[3]{-\frac{q}{2} - \sqrt{R}} = \sqrt[3]{-\left(-\frac{1}{2} : 2\right) + \frac{5\sqrt{3}}{36}} + \sqrt[3]{-\left(-\frac{1}{2} : 2\right) - \frac{5\sqrt{3}}{36}} =$$

$$= \sqrt[3]{\frac{1}{4} + \frac{5\sqrt{3}}{36}} + \sqrt[3]{\frac{1}{4} - \frac{5\sqrt{3}}{36}} = \boxed{\sqrt[3]{\frac{9 + 5\sqrt{3}}{36}} + \sqrt[3]{\frac{9 - 5\sqrt{3}}{36}}}$$

First section of handwritten text, appearing as a short paragraph or list of items.

Second section of handwritten text, continuing the narrative or list.

Third section of handwritten text, possibly a separate entry or a continuation.

Fourth section of handwritten text, located near the bottom of the main body.

mas teniendo en cuenta el valor de " x ," ya obtenido anteriormente, podemos escribir la notable propiedad numérica siguiente:

$$\sqrt[3]{\frac{9+5\sqrt{3}}{36}} + \sqrt[3]{\frac{9-5\sqrt{3}}{36}} = 1$$

$$\cos \beta = 1$$

$$\beta = 0^\circ$$

Las dos raíces restantes son imaginarias

1.º Caso límite en que $\frac{a}{b} = 0$

En este caso será $l = \frac{a}{b} = 0$

en la ecuación cúbica (1), tendremos

$$\frac{a}{8l} = \frac{0}{8} = 0$$

por lo que aquella se transformará en

$$x^3 - \frac{1}{2}x - 0 = 0$$

$$0 \text{ sea } \left. \begin{array}{l} x^3 = \frac{1}{2}x \quad x^2 = \frac{1}{2} \quad x_2 = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \\ x_3 = -\sqrt{\frac{1}{2}} = -\frac{\sqrt{2}}{2} \end{array} \right\} \begin{array}{l} \cos \beta = \frac{\frac{\sqrt{2}}{2}}{1} \\ \beta = 45^\circ \end{array}$$

Si aplicamos la fórmula general de Cardano, tendremos, haciendo

$$p = -\frac{1}{2} \quad q = 0$$

$$R = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 = \left(-\frac{1}{2 \times 3}\right)^3 = -\frac{1}{2^3 \times 3^3} \quad \text{y de aquí}$$

$$\sqrt{R} = \sqrt{-\frac{1}{2^3 \times 3^3}} = \frac{1}{6} \times \sqrt{-\frac{1}{6}} = \frac{1}{6} \times \frac{\sqrt{6}}{6} i = \frac{\sqrt{6}}{36} i$$

Write the following in Hindi -
1. Write the name of the country which is the largest in the world.

2. Write the name of the country which is the smallest in the world.

3. Write the name of the country which is the most populous in the world.

4. Write the name of the country which is the least populous in the world.

5. Write the name of the country which is the most developed in the world.

6. Write the name of the country which is the least developed in the world.

por lo que será

$$x_1 = \sqrt[3]{-\frac{9}{2} + \sqrt{R}} + \sqrt[3]{-\frac{9}{2} - \sqrt{R}} = \sqrt[3]{\frac{\sqrt{6}}{36} i} + \sqrt[3]{-\frac{\sqrt{6}}{36} i} = -\sqrt[3]{\frac{\sqrt{6}}{36}} + \sqrt[3]{\frac{\sqrt{6}}{36}} = 0$$

$$\cos \beta = 0$$

$$\beta = 90^\circ$$

En el problema planteado sólo tiene sentido geométrico la raíz " x_2 "

$$\cos \beta = \frac{\sqrt{2}}{2} \quad \beta = 45^\circ$$

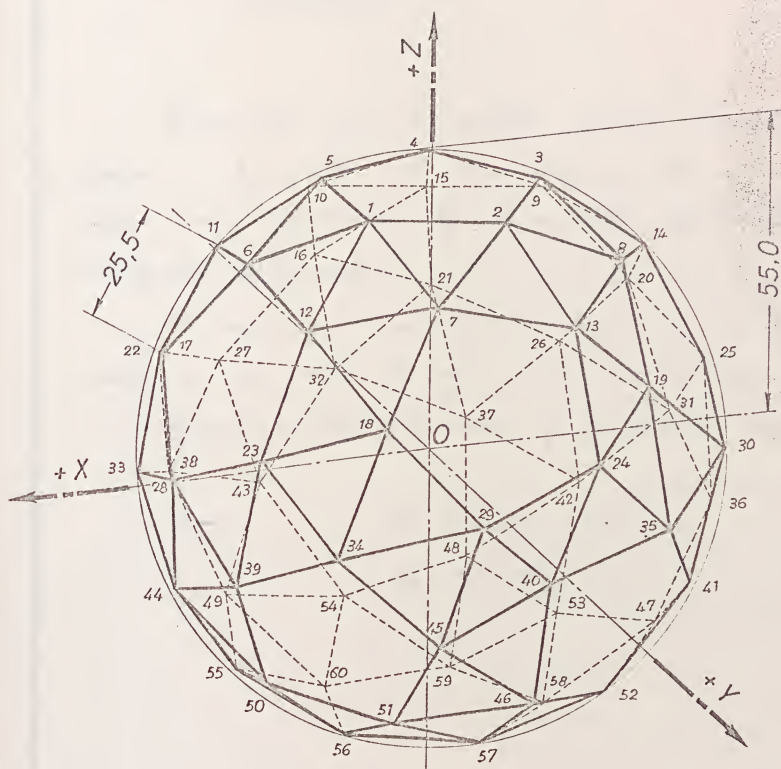
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Arquimedeano II

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimédiano III en el que en cada vértice concurren dos triángulos equiláteros y dos cuadrados, alternados.

La longitud de su lado es de 55 mm. y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3V. y a escala 1:1

DATOS

O (72, 72, 85) mm

 $l_{III} = 55 \text{ mm.}$

The first of the two papers in this section is by Professor J. H. J. van der Linde, who discusses the role of the 'tribe' in the formation of the 'state' in South Africa. He argues that the 'tribe' is a social formation which is based on kinship and which is characterized by a high degree of internal cohesion and a low degree of external cohesion. He suggests that the 'tribe' is a social formation which is based on kinship and which is characterized by a high degree of internal cohesion and a low degree of external cohesion.

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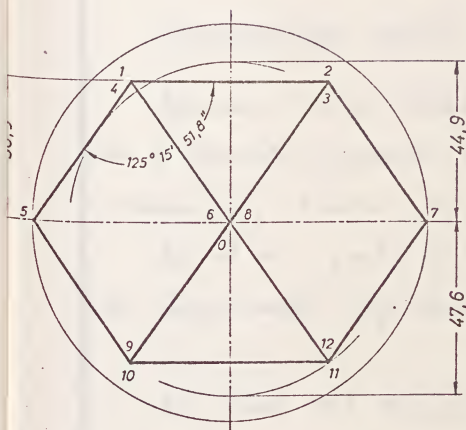
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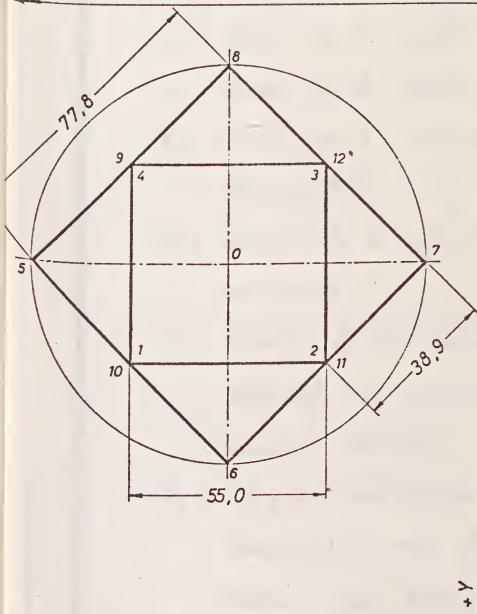
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I

+Z

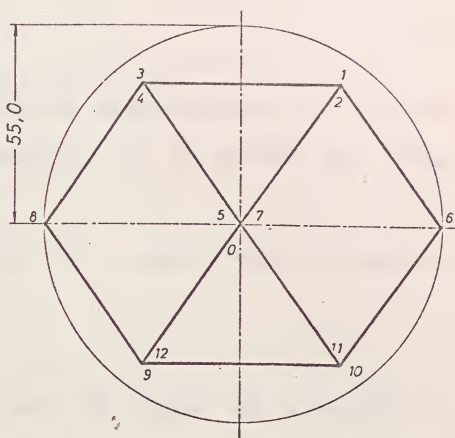


+X



+Y

III



ARQUIMEDIANO III

Número de caras triangulares..... $C_3 = 8$
 Número de caras cuadradas..... $C_4 = 6$
 Número de vértices..... $V = 12$
 Número de aristas..... $A = 24$
 Número de caras de un ángulo sólido: $2C_3 + 2C_4$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimedeano III, en el que en cada vértice concurren dos triángulos equiláteros y dos cuadrados.

La longitud de su lado es de 55 milímetros y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

II

	Propuesta	De entrega	Entregada	Calificación		Escuela
Fecha:					(firma)	Curso
Alumno:						
Escala	Arquimedeano III					
1:1						

Lámina 35

Curso 19 - 19



Diagram illustrating the construction of a hexagon from a square.

Let $ABCD$ be a square. Construct the midpoints E, F, G, H of the sides AB, BC, CD, DA respectively. Connect E, F, G, H to form an inner square $EFGH$. The lines AC and BD intersect at O . The lines EF, FG, GH, HE intersect AC and BD at points P, Q, R, S respectively. The hexagon $OPQR$ is the desired construction.

Diagram illustrating the construction of a hexagon from a square.

Let $ABCD$ be a square. Construct the midpoints E, F, G, H of the sides AB, BC, CD, DA respectively. Connect E, F, G, H to form an inner square $EFGH$. The lines AC and BD intersect at O . The lines EF, FG, GH, HE intersect AC and BD at points P, Q, R, S respectively. The hexagon $OPQR$ is the desired construction.



CONSIDERACIONES PREVIAS

Seguiremos en el estudio de este arquimediano, las directrices y fórmulas generales planteadas en el estudio del "Arquimediano I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

l = Arista del Arquimediano III (dato del ejercicio)

a = Radio de la esfera circunscrita

b = Radio de la esfera tangente a las aristas

c_3 = Radio de la esfera tangente a las caras triangulares

c_4 = Radio de la esfera tangente a las caras cuadradas.

d_3 = Radio de la circunferencia circunscrita a una cara triangular.

d_4 = Radio de la circunferencia circunscrita a una cara cuadrada.

m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

α_3 = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimediano, que pasa por una arista de aquella.

α_4 = Ángulo rectilíneo del diedro formado por una cara cuadrada, con el plano diametral del arquim-

mediano, que pasa por una arista de aquella.

$f_{3,4}$ = Ángulo rectilíneo del diedro formado por una cara triangular y otra cuadrada.

S = Superficie

V = Volumen

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimediano, nos indica que tiene 8 caras regulares triangulares, y 6 caras cuadradas; 12 vértices y 24 aristas.

En cada vértice concurren 2 caras triangulares, 2 cuadradas y, por consiguiente, 4 aristas del mismo.

Así pues, tendremos que

$$\text{Arquimediano III } (2P_3 + 2P_4); C_3 = 8; C_4 = 6; V = 12; A = 24$$

Cálculo de sus magnitudes

Arista "l" del arquimediano

Dato del ejercicio

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las cuatro aristas de un án-

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<p> Witness my hand and seal of office this day of 19 </p>		
<p> (Signature of Clerk) </p>		
<p> I hereby certify that the within copy is a true and correct copy of the original as the same appears on the records of the County of State of </p>		

que sólido.

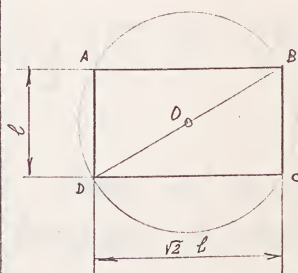


Figura 1

Este polígono es un rectángulo ABCD (fig. 1), uno de cuyos lados es el lado del arquimedianos (lado de las caras triangulares) y el otro la diagonal de las caras cuadradas.

El radio pedido será la semi-diagonal OD de dicho rectángulo.

Su valor será pues

$$\overline{OD} = \boxed{m} = \frac{1}{2} \sqrt{l^2 + (\sqrt{3}l)^2} = \frac{\sqrt{3}}{2} l = 0.8660254... l$$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33) a este caso particular de "m"

$$\boxed{a} = \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - (\frac{\sqrt{3}}{2}l)^2}} = \boxed{l}$$

Desarrollo del cálculo anterior:

$$\boxed{a} = \frac{l^2}{2\sqrt{l^2 - (\frac{\sqrt{3}}{2}l)^2}} = \frac{l^2}{2\sqrt{l^2 - \frac{3}{4}l^2}} = \frac{l^2}{2\sqrt{1 - \frac{3}{4}} l} = \frac{l}{2\sqrt{\frac{1}{4}}} = l$$

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 the references.



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The results of the experiment
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 table. The first column shows
 the time taken for the
 reaction to occur. The second
 column shows the amount of
 product formed.

$$\text{Reaction 1: } \text{A} + \text{B} \rightarrow \text{C} + \text{D}$$

The results of the experiment
 are shown in the following
 table. The first column shows
 the time taken for the
 reaction to occur. The second
 column shows the amount of
 product formed.

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3] (ver lám. 33), tendremos:

$$\boxed{b} = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{l^2 - \frac{l^2}{4}} = \sqrt{\frac{3l^2}{4}} = \frac{\sqrt{3}}{2} l = 0.8660254... l$$

Radio "d₃" de la circunferencia circunscrita a una cara triangular de lado "l"

Se demuestra en Geometría, es

$$\boxed{d_3} = \frac{\sqrt{3}}{3} l = 0.57735027... l$$

Radio "d₄" de la circunferencia circunscrita a una cara cuadrada de lado "l"

Se demuestra en Geometría, es

$$\boxed{d_4} = \frac{\sqrt{2}}{2} l = 0.7071068... l$$

Radio "c₃" de la esfera tangente a las caras triangulares regulares de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\boxed{c_3} = \sqrt{a^2 - (d_3)^2} = \sqrt{l^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{2}{3}} l = \frac{\sqrt{6}}{3} l = 0.81649658... l$$

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Radio " c_4 " de la esfera tangente a las caras cuadradas de lado " l "

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\boxed{c_4} = \sqrt{a^2 - (d_4)^2} = \sqrt{l^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} = \sqrt{1 - \frac{1}{2}} \cdot l = \boxed{\frac{\sqrt{2}}{2} l} = 0.7071068... l$$

Ángulo rectilíneo " α_3 " del diedro formado por una cara triangular, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

Se determina, en función de su tangente, por la fórmula general [5] (ver lám. 33):

$$\boxed{\tan \alpha_3} = \frac{2 c_3}{\sqrt{4 (d_3)^2 - l^2}} = \frac{2 \times \frac{\sqrt{6}}{3} l}{\sqrt{4 \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \boxed{2 \sqrt{2}} = 2.82842712...$$

Desarrollo del cálculo anterior:

$$\boxed{\tan \alpha_3} = \frac{2 \times \frac{\sqrt{6}}{3} l}{\sqrt{4 \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \frac{\frac{2 \sqrt{6}}{3}}{\sqrt{\frac{4}{3} - 1}} = \frac{2 \sqrt{6}}{3} : \frac{1}{\sqrt{3}} = \frac{2 \sqrt{18}}{3} = \frac{2 \times 3 \sqrt{2}}{3} = \boxed{2 \sqrt{2}}$$

$$\text{y } \frac{1}{\tan \alpha_3} = 2.82842712 = 0.4515450$$

$$\boxed{\alpha_3 = 70^\circ 31' 43.6''}$$

Ángulo rectilíneo " α_4 " del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

Subject: _____

Topic: _____

Q. No. _____

Sol. _____

Se determina, en función de su tangente, por la fórmula general [6] (ver lám. 33)

$$\tan \alpha_4 = \frac{2 c_4}{\sqrt{4 (d_4)^2 - l^2}} = \frac{2 \times \frac{\sqrt{2}}{2} l}{\sqrt{4 \left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} = \boxed{\sqrt{2}} = 1,41421356 \dots$$

Desarrollo del cálculo anterior:

$$\tan \alpha_4 = \frac{2 \times \frac{\sqrt{2}}{2} l}{\sqrt{4 \left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} = \frac{\sqrt{2} l}{\sqrt{(2-1) l^2}} = \boxed{\sqrt{2}}$$

$$\log \tan \alpha_4 = \log 1,41421356 = 0,1505150$$

$$\alpha_4 = 54^\circ 44' 8,2''$$

Ángulo rectilíneo " φ_{3-4} " del diedro formado por una cara triangular y una cuadrada

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\begin{aligned} \boxed{\varphi_{3-4}} &= \alpha_3 + \alpha_4 = 70^\circ 31' 43,6'' + 54^\circ 44' 8,2'' = \\ &= \boxed{125^\circ 15' 51,8''} \end{aligned}$$

Comprobación:

Siendo $\varphi_{3-4} = \alpha_3 + \alpha_4$, tendremos que

$$\tan \varphi_{3-4} = \frac{\tan \alpha_3 + \tan \alpha_4}{1 - \tan \alpha_3 \tan \alpha_4} = \frac{2\sqrt{2} + \sqrt{2}}{1 - 2\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{1-4} = -\sqrt{2} = -\tan \alpha_4$$

7 por lo tanto:

$$\varphi_{3-4} + \alpha_4 = 125^\circ 15' 51,8'' + 54^\circ 44' 8,2'' = 180^\circ \text{ (suplementarios)}$$

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Área lateral "S" del arquimediano

Se compone de 8 caras triangulares y 6 cuadradas, de lado "l"; la superficie total será:

$$\boxed{S} = 8 \times \frac{\sqrt{3}}{4} l^2 + 6 l^2 = (2\sqrt{3} + 6) l^2 = \boxed{2(\sqrt{3} + 3) l^2} = 9.46410162... l^2$$

Volumen "V" del arquimediano

Se compone de la suma de 8 pirámides de base triangular y altura "c₃" y de 6 pirámides de base cuadrada y altura "c₄"; su valor será:

$$\boxed{V} = 8 \times \frac{\sqrt{3}}{4} l^2 \times \frac{c_3}{3} + 6 l^2 \times \frac{c_4}{3} = \frac{2\sqrt{3}}{3} l^2 \times \frac{\sqrt{6}}{3} l + 2 l^2 \times \frac{\sqrt{2}}{2} l =$$

$$= \boxed{\frac{5\sqrt{2}}{3} l^3} = 2.35702260... l^3$$

Desarrollo del cálculo anterior,

$$V = \frac{2\sqrt{3}}{3} l^2 \times \frac{\sqrt{6}}{3} l + 2 l^2 \times \frac{\sqrt{2}}{2} l = \left(\frac{2\sqrt{18}}{9} + \sqrt{2} \right) l^3 = \left(\frac{2\sqrt{2}}{3} + \sqrt{2} \right) l^3 =$$

$$= \frac{5\sqrt{2}}{3} l^3$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 8 triángulos equiláteros y

6 cuadrados, de lado 55 mm, de forma que en cada vértice concurren 2 triángulos y 2 cuadrados alternados.

A continuación damos un resumen tabulado de las anteriores magnitudes calculadas.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	l	1.00 00 00... l
b	$\frac{\sqrt{3}}{2} l$	0.86 60 25... l
c_3	$\frac{\sqrt{6}}{3} l$	0.81 64 97... l
c_4	$\frac{\sqrt{2}}{2} l$	0.70 71 07... l
d_3	$\frac{\sqrt{3}}{3} l$	0.57 73 50... l
d_4	$\frac{\sqrt{2}}{2} l$	0.70 71 07... l
m	$\frac{\sqrt{3}}{2} l$	0.86 60 25... l
α_3	$\operatorname{tg} \alpha_3 = 2\sqrt{2}$	$\operatorname{tg} \alpha_3 = 2.82 84 27...$ $\alpha_3 = 70^\circ 31' 43.6''$
α_4	$\operatorname{tg} \alpha_4 = \sqrt{2}$	$\operatorname{tg} \alpha_4 = 1.41 42 14...$ $\alpha_4 = 54^\circ 44' 8.2''$
φ_{3-4}	$\alpha_3 + \alpha_4$	$\varphi_{3-4} = 125^\circ 15' 51.8''$
S	$2(\sqrt{3} + 3) l^2$	9.46 41 02... l^2
V	$\frac{5\sqrt{2}}{3} l^3$	2.35 70 23... l^3
Relaciones entre magnitudes		
$a = l \qquad b = m \qquad c_4 = d_4$		

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de magnitudes principales, vamos a proceder en la lámina 35, a la representación del Arquimediano III cuyo lado conocido, es de 55 mm.

Calculamos previamente las siguientes magnitudes:

$$l_{III} = (\text{dato del ejercicio}) = 55,0 \text{ mm}$$

$$\alpha = l_{III} = 55,0 \text{ mm}$$

$$b = 0,866025 \dots \times 55 = 47,6 \text{ mm}$$

$$C_3 = 0,816497 \dots \times 55 = 44,9 \text{ mm}$$

$$C_4 = 0,707107 \dots \times 55 = 38,9 \text{ mm}$$

$$d_3 = 0,577350 \dots \times 55 = 31,7 \text{ mm}$$

$$d_4 = 0,707107 \dots \times 55 = 38,9 \text{ mm}$$

El orden de operaciones para el trazado gráfico, es el siguiente:

1° Situar el centro O de coordenadas 72, 72, 85.

2° Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio $a = 55 \text{ mm}$.

3° Representar en I, II y III la cara cuadrada 1-2-3-4, supuesto el poliedro colocado con dicha cara paralela a II y un lado (1-4) perpendicular a I. (utilícese la cota " C_6 " en I y III).

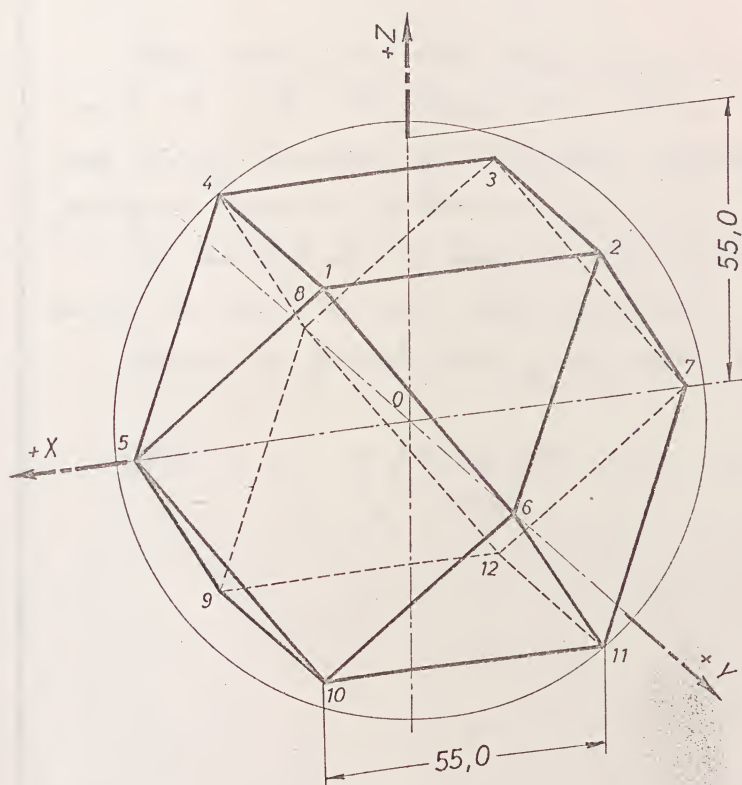
4° Trazar en II el cuadrado 5-6-7-8, circunscrito al 1-2-3-4 (lados perpendiculares a las diagonales).

Name	Address
John Doe	123 Main St, New York, NY
Jane Smith	456 Elm St, New York, NY
Robert Brown	789 Oak St, New York, NY
Mary White	101 Pine St, New York, NY
James Green	202 Cedar St, New York, NY
Elizabeth Black	303 Birch St, New York, NY
William Gray	404 Spruce St, New York, NY
Margaret Hall	505 Willow St, New York, NY
Richard King	606 Ash St, New York, NY
Susan Lee	707 Hickory St, New York, NY
Thomas Scott	808 Sycamore St, New York, NY
Patricia Young	909 Magnolia St, New York, NY
Daniel Hill	1010 Poplar St, New York, NY
Jennifer Adams	1111 Cherry St, New York, NY

5° Con el trazado ya realizado, puede terminarse la representación en I y III, por simetría axial, pudiendo observarse que son iguales de forma, pero no en lo que respecta a la colocación de vértices

6° Numerar los vértices y comprobar en el dibujo las magnitudes " b ", " c_3 " y " $d_4 = c_4$ ".





Arquimediano III



Fig. 1. Crystal structure of the compound.

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimedianos IV en el que en cada vértice concurren dos triángulos equiláteros y dos pentágonos regulares, alternados.

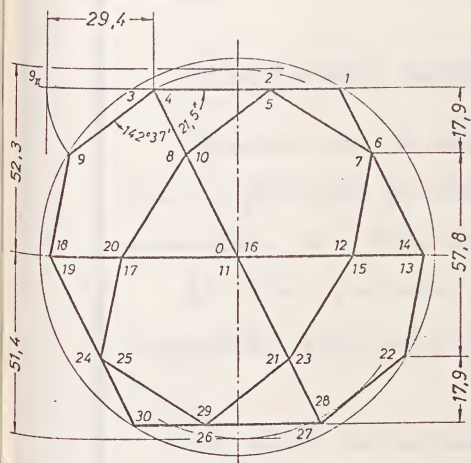
La longitud de su lado es 34 mm y las coordenadas de su centro O, son $O(72, 72, 85)$ mm.

Dibujar en formato A3v y a escala 1:1

DATOS

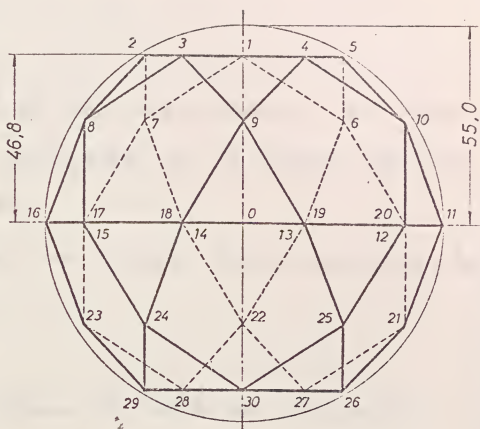
 $O(72, 72, 85)$ mm $l_N = 34$ mm

I



+Z

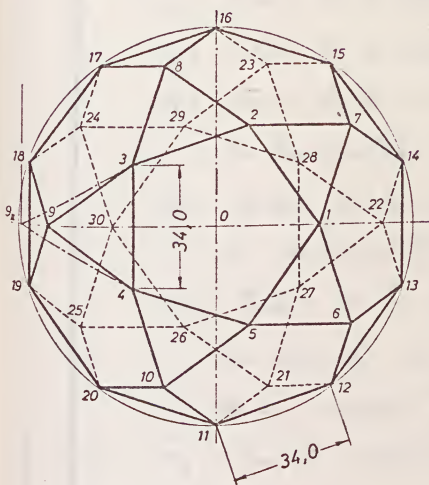
III



+X

O

+Y



+Y

ARQUIMEDIANO IV

Número de caras triangulares..... $C_3 = 20$
 Número de caras pentagonales..... $C_5 = 12$
 Número de vértices..... $V = 30$
 Número de aristas..... $A = 60$
 Número de caras de un ángulo sólido: $2C_3 + 2C_5$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimiliano IV, en el que en cada vértice concurren dos triángulos equiláteros y dos pentágonos regulares.

La longitud de su lado es de 34 milímetros y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

II

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Arquimiliano IV				Lámina 36
1:1					Curso 19 - 19



PROPOSITION 12.
 If a circle be described about an equilateral triangle, the square of the radius is equal to the square of the side of the triangle divided by three.

Let ABC be an equilateral triangle, and let a circle be described about it, so that the center of the circle may be the same as the center of the triangle. Let D be the center of the circle, and let DE be a radius drawn to the point E on the circumference. Let DF be a line drawn from the center D to the side AC, meeting AC at F. Then DF is perpendicular to AC, and AF is equal to FC. Since ABC is an equilateral triangle, the angle BAC is equal to the angle ABC, and each is equal to 60 degrees. Since D is the center of the circle, the angle ADE is equal to the angle BDE, and each is equal to 120 degrees. Since DE is a radius, and DF is a line drawn from the center to the side AC, the angle EDF is equal to 60 degrees. Therefore, the triangle EDF is a right-angled triangle, with the right angle at F. Hence, the square of the radius DE is equal to the square of the side EF plus the square of the side DF. But EF is equal to half the side AC, and DF is equal to the distance from the center of the circle to the side AC. Therefore, the square of the radius is equal to the square of half the side of the triangle plus the square of the distance from the center of the circle to the side of the triangle. This is the same as saying that the square of the radius is equal to the square of the side of the triangle divided by three.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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CONSIDERACIONES PREVIAS

Seguiremos en el estudio de este arquimediano, las directrices y fórmulas generales planteadas en el estudio del "Arquimediano I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

l = Arista del arquimediano IV. (dato del ejercicio)

a = Radio de la esfera circunscrita.

b = Radio de la esfera tangente a las aristas.

c_3 = Radio de la esfera tangente a las caras triangulares.

c_5 = Radio de la esfera tangente a las caras pentagonales.

d_3 = Radio de la circunferencia circunscrita a una cara triangular.

d_5 = Radio de la circunferencia circunscrita a una cara pentagonal.

m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

α_3 = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimediano, que pasa por una arista de aquélla.

α_5 = Ángulo rectilíneo del diedro formado por una cara pentagonal regular, con el plano diametral del

arquimedeo, que pasa por una arista de aquella.

φ_{3-5} = Ángulo acutángulo del diedro formado por una cara triangular y otra pentagonal.

S = Superficie

V = Volumen

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedeo, nos indica que se compone de 20 caras triangulares regulares, y 12 caras pentagonales regulares; 30 vértices y 60 aristas.

En cada vértice concurren alternadamente 2 caras triangulares y dos pentagonales y, por consiguiente, 4 aristas del mismo.

Así pues, tendremos que

$$\text{ARQUIMEDEO IV } (2 P_3 + 2 P_5); C_3 = 20; C_5 = 12; V = 30; A = 60$$

Cálculo de sus magnitudes

Arista "l" del arquimedeo

Dato del ejercicio



Received of the Treasurer of the
Board of Directors of the
City of New York
the sum of \$100.00
for the year 1900

Witness my hand and the seal of the
City of New York this 1st day of
January 1901

Mayor of the City of New York

John A. B. Smith
Secretary of the Board of Directors

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las cuatro aristas de un ángulo sólido.

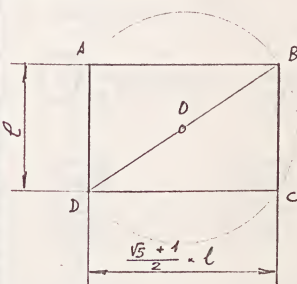


Figura 1

Este polígono es el rectángulo A.B.C.D (fig. 1), uno de cuyos lados es el lado del arquimedianos (lado de las caras triangulares) y el otro es la diagonal del pentágono de lado "l" de su cara continua.

El valor de la diagonal del pentágono regular, en función de su lado, se demuestra en Geometría es

$$DC = \frac{\sqrt{5} + 1}{2} l$$

El radio pedido será la semidiagonal OD de dicho rectángulo. Su valor será pues

$$OD = \boxed{m} = \frac{1}{2} \sqrt{l^2 + \left(\frac{\sqrt{5}+1}{2} l\right)^2} = \boxed{\sqrt{\frac{5+\sqrt{5}}{8}} l} = 0,95105651... l$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \boxed{m} &= \frac{1}{2} \sqrt{l^2 + \left(\frac{\sqrt{5}+1}{2} l\right)^2} = \frac{1}{2} \sqrt{l^2 + \frac{(\sqrt{5}+1)^2}{4} l^2} = \frac{1}{2} \sqrt{1 + \frac{5+1+2\sqrt{5}}{4}} l = \\ &= \frac{1}{2} \sqrt{1 + \frac{6+2\sqrt{5}}{4}} l = \frac{1}{2} \sqrt{1 + \frac{3+\sqrt{5}}{2}} l = \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{2}} l = \boxed{\sqrt{\frac{5+\sqrt{5}}{8}} l} \end{aligned}$$

The first part of the problem is to find the area of the rectangle. The area of a rectangle is given by the formula $A = l \times b$, where l is the length and b is the breadth.

In this case, the length is 12 cm and the breadth is 8 cm. Therefore, the area is $12 \times 8 = 96$ cm².



The second part of the problem is to find the perimeter of the rectangle. The perimeter of a rectangle is given by the formula $P = 2(l + b)$, where l is the length and b is the breadth.

In this case, the length is 12 cm and the breadth is 8 cm. Therefore, the perimeter is $2(12 + 8) = 40$ cm.

The third part of the problem is to find the area of the square. The area of a square is given by the formula $A = s^2$, where s is the side length.

In this case, the side length is 10 cm. Therefore, the area is $10^2 = 100$ cm².

The fourth part of the problem is to find the perimeter of the square. The perimeter of a square is given by the formula $P = 4s$, where s is the side length.

In this case, the side length is 10 cm. Therefore, the perimeter is $4 \times 10 = 40$ cm.

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33), a este caso particular.

$$a = \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - \left(\frac{\sqrt{5+15}}{8}l\right)^2}} = \frac{l}{2\sqrt{1 - \frac{5+15}{64}}} = \frac{l}{2\sqrt{\frac{3-15}{8}}} =$$

$$= \frac{\sqrt{\frac{3-15}{8}}}{\frac{3-15}{8}} l = \frac{4\sqrt{\frac{3-15}{8}}}{3-15} l = \frac{4\sqrt{\frac{3-15}{8}} \times (3+15)}{4} l = \sqrt{\frac{(3-15)(3+15)^2}{8}} l =$$

$$= \sqrt{\frac{4(3+15)}{8}} l = \sqrt{\frac{3+15}{2}} l = \frac{\sqrt{3+15}}{\sqrt{2}} l = \frac{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}}{\sqrt{2}} l = \boxed{\frac{\sqrt{5}+1}{2} l} =$$

1. 61 80 33 99... l (igual a la diagonal de una casa pentagonal)

Radio "b" de la esfera tangente a las aristas.

Aplicando la fórmula general [3] (ver lám. 33), tendremos:

$$b = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{\sqrt{3+15}}{2}l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{3+15}{4} - \frac{1}{4}} l = \sqrt{\frac{6+2\sqrt{5}-1}{4}} l =$$

$$= \sqrt{\frac{5+2\sqrt{5}}{4}} l = \boxed{\frac{\sqrt{5+2\sqrt{5}}}{2} l} = 1.53 88 41 76... l$$

Radio "d₃" de la circunferencia circunscrita a una triángulo de lado l

Se demuestra en Geometría, es

$$\boxed{d_3 = \frac{\sqrt{3}}{3} l} = 0.57 73 50 27... l$$

The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$
 and to the determination of its asymptotic behavior as $x \rightarrow 0$ and $x \rightarrow \infty$. It is shown that the function $f(x)$ is bounded and continuous on the interval $(0, \infty)$ and that it satisfies the differential equation

$$x^2 f''(x) + x f'(x) - f(x) = 0$$
 which can be solved by the method of separation of variables. The general solution of this equation is

$$f(x) = C_1 \sqrt{x} + C_2 \frac{1}{\sqrt{x}}$$
 where C_1 and C_2 are arbitrary constants. The boundary conditions $f(0) = 0$ and $f(\infty) = 0$ lead to the conclusion that $C_1 = 0$ and $C_2 = 0$, which implies that $f(x) = 0$ for all $x > 0$. This result is in contradiction with the assumption that $f(x)$ is not identically zero. Therefore, the only solution of the problem is $f(x) = 0$.

los " d_5 " de la circunferencia circunscrita a una cara regular.

Se demuestra en Geometría es

$$d_5 = \sqrt{\frac{5 + \sqrt{5}}{10}} l = 0, 85 06 50 8 \dots l$$

Radio " c_3 " de la esfera tangente a las caras triangulares regulares de lado " l "

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$c_3 = \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\frac{\sqrt{5}+1}{2} l\right)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{3+\sqrt{5}}{2} - \frac{1}{3}} \cdot l =$$

$$\sqrt{\frac{9+3\sqrt{5}-2}{6}} l = \sqrt{\frac{7+3\sqrt{5}}{6}} l = \frac{\sqrt{7+3\sqrt{5}}}{\sqrt{6}} l = \frac{\sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}}}{\sqrt{6}} l = \left(\sqrt{\frac{9}{12}} + \sqrt{\frac{5}{12}}\right) l =$$

$$\left(\frac{3}{\sqrt{12}} + \frac{\sqrt{5}}{2\sqrt{3}}\right) l = \left(\frac{3\sqrt{3}}{6} + \frac{\sqrt{15}}{6}\right) l = \frac{3\sqrt{3} + \sqrt{15}}{6} l = 1, 51 15 22 63 \dots l$$

Radio " c_5 " de la esfera tangente a las caras pentagonales de lado " l "

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$c_5 = \sqrt{a^2 - (d_5)^2} = \sqrt{\left(\frac{\sqrt{5}+1}{2} l\right)^2 - \left(\sqrt{\frac{5+\sqrt{5}}{10}} l\right)^2} = \sqrt{\frac{3+\sqrt{5}}{2} - \frac{5+\sqrt{5}}{10}} \cdot l =$$

$$\sqrt{\frac{5+5\sqrt{5}-5-\sqrt{5}}{10}} l = \sqrt{\frac{10+4\sqrt{5}}{10}} l = \sqrt{\frac{5+2\sqrt{5}}{5}} l = 1, 37 63 81 9 \dots l$$

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations. The second part is devoted to the construction of the solution. It is shown that the solution can be obtained by the method of variation of parameters. The third part is devoted to the study of the properties of the solution. It is shown that the solution is unique and stable. The fourth part is devoted to the application of the results to the theory of differential equations. It is shown that the results can be applied to the study of the stability of the equilibrium point.

Ángulo rectilíneo " α_3 " del diedro formado por una cara triangular, con el plano diametral del arquimediano que pasa por una arista de aquélla.

Se determina, en función de su tangente, por la fórmula general [5] (ver lám. 33)

$$\begin{aligned} \boxed{t_{\phi} \alpha_3} &= \frac{2c_3}{\sqrt{4(d_3)^2 - l^2}} = \frac{2 \times \frac{3\sqrt{3} + \sqrt{15}}{6} l}{\sqrt{4 \times \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \frac{3\sqrt{3} + \sqrt{15}}{3\sqrt{4 \times \frac{1}{3} - 1}} = \\ &= \frac{3\sqrt{3} + \sqrt{15}}{3\sqrt{\frac{1}{3}}} = \frac{3\sqrt{3} + \sqrt{15}}{\frac{3}{\sqrt{3}}} = \frac{9 + \sqrt{45}}{3} = \frac{9 + 3\sqrt{5}}{3} = \boxed{3 + \sqrt{5}} = 5, 23\ 60\ 67\ 98. \end{aligned}$$

$$\hookrightarrow t_{\phi} \alpha_3 = 0,719\ 00\ 52$$

$$\boxed{\alpha_3 = 49^\circ\ 11'\ 15,7''}$$

Ángulo rectilíneo " α_5 " del diedro formado por una cara pentagonal, con el plano diametral del arquimediano que pasa por una arista de aquélla.

Se determina, en función de la tangente, por la fórmula general [6] (ver lám. 33).

$$\begin{aligned} \boxed{t_{\phi} \alpha_5} &= \frac{2c_5}{\sqrt{4(d_5)^2 - l^2}} = \frac{2\sqrt{\frac{5+2\sqrt{5}}{5}} l}{\sqrt{4\left(\sqrt{\frac{5+\sqrt{5}}{10}} l\right)^2 - l^2}} = \frac{2\sqrt{\frac{5+2\sqrt{5}}{5}}}{\sqrt{4 \times \frac{5+\sqrt{5}}{10} - 1}} = \frac{2\sqrt{\frac{5+2\sqrt{5}}{5}}}{\sqrt{\frac{10+2\sqrt{5}}{5} - 1}} = \\ &= \frac{2\sqrt{\frac{5+2\sqrt{5}}{5}}}{\sqrt{\frac{5+2\sqrt{5}}{5}}} = 2\sqrt{\frac{5+2\sqrt{5}}{5} \cdot \frac{5+2\sqrt{5}}{5}} = \boxed{2} \end{aligned}$$

$$\hookrightarrow t_{\phi} \alpha_5 = t_{\phi} 2 = 0,30\ 10\ 30\ 0$$

$$\boxed{\alpha_5 = 63^\circ\ 26'\ 5,8''}$$

The first part of the problem is to find the value of x such that $x^2 + 1 = 0$. This is a quadratic equation, and we can solve it by using the quadratic formula. The quadratic formula is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In this case, $a = 1$, $b = 0$, and $c = 1$. Substituting these values into the formula, we get $x = \frac{0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{0 \pm \sqrt{-4}}{2} = \frac{0 \pm 2i}{2} = \pm i$. Therefore, the solutions to the equation $x^2 + 1 = 0$ are $x = i$ and $x = -i$.

$$\begin{aligned}
 & \frac{d}{dx} \left(\frac{x^2 + 1}{x^2 - 1} \right) = \frac{(x^2 - 1) \cdot \frac{d}{dx}(x^2 + 1) - (x^2 + 1) \cdot \frac{d}{dx}(x^2 - 1)}{(x^2 - 1)^2} \\
 & = \frac{(x^2 - 1) \cdot 2x - (x^2 + 1) \cdot 2x}{(x^2 - 1)^2} = \frac{2x(x^2 - 1) - 2x(x^2 + 1)}{(x^2 - 1)^2} \\
 & = \frac{2x(x^2 - 1 - x^2 - 1)}{(x^2 - 1)^2} = \frac{2x(-2)}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}
 \end{aligned}$$

The second part of the problem is to find the value of y such that $y^2 + 1 = 0$. This is a quadratic equation, and we can solve it by using the quadratic formula. The quadratic formula is given by $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In this case, $a = 1$, $b = 0$, and $c = 1$. Substituting these values into the formula, we get $y = \frac{0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{0 \pm \sqrt{-4}}{2} = \frac{0 \pm 2i}{2} = \pm i$. Therefore, the solutions to the equation $y^2 + 1 = 0$ are $y = i$ and $y = -i$.

$$\begin{aligned}
 & \frac{d}{dy} \left(\frac{y^2 + 1}{y^2 - 1} \right) = \frac{(y^2 - 1) \cdot \frac{d}{dy}(y^2 + 1) - (y^2 + 1) \cdot \frac{d}{dy}(y^2 - 1)}{(y^2 - 1)^2} \\
 & = \frac{(y^2 - 1) \cdot 2y - (y^2 + 1) \cdot 2y}{(y^2 - 1)^2} = \frac{2y(y^2 - 1) - 2y(y^2 + 1)}{(y^2 - 1)^2} \\
 & = \frac{2y(y^2 - 1 - y^2 - 1)}{(y^2 - 1)^2} = \frac{2y(-2)}{(y^2 - 1)^2} = \frac{-4y}{(y^2 - 1)^2}
 \end{aligned}$$

The third part of the problem is to find the value of z such that $z^2 + 1 = 0$. This is a quadratic equation, and we can solve it by using the quadratic formula. The quadratic formula is given by $z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In this case, $a = 1$, $b = 0$, and $c = 1$. Substituting these values into the formula, we get $z = \frac{0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{0 \pm \sqrt{-4}}{2} = \frac{0 \pm 2i}{2} = \pm i$. Therefore, the solutions to the equation $z^2 + 1 = 0$ are $z = i$ and $z = -i$.

Ángulo rectilíneo " φ_{3-5} " del diedro formado por una cara triangular y una pentagonal regulares.

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{3-5}} = \alpha_3 + \alpha_5 = 79^\circ 11' 15,7'' + 63^\circ 26' 58'' =$$

$$= \boxed{142^\circ 37' 21,5''}$$

COMPROBACION

$$\begin{aligned} \tan \varphi_{3-5} &= \tan (\alpha_3 + \alpha_5) = \frac{\tan \alpha_3 + \tan \alpha_5}{1 - \tan \alpha_3 \tan \alpha_5} = \frac{(3 + \sqrt{5}) + 2}{1 - (3 + \sqrt{5}) \times 2} = \\ &= \frac{5 + \sqrt{5}}{1 - 6 - 2\sqrt{5}} = \frac{5 + \sqrt{5}}{-5 - 2\sqrt{5}} = - \frac{5 + \sqrt{5}}{5 + 2\sqrt{5}} = - \frac{(5 + \sqrt{5})(5 - 2\sqrt{5})}{5} = \\ &= - \frac{25 + 5\sqrt{5} - 10\sqrt{5} - 10}{5} = - \frac{15 - 5\sqrt{5}}{5} = - (3 - \sqrt{5}) = - 0,76373202... \\ \therefore \tan \varphi_{3-5} &= \tan 0,76373202 = \bar{1},8830518 \\ -\varphi_{3-5} &= -37^\circ 22' 38,5'' \end{aligned}$$

$$\boxed{\varphi_{3-5}} = 180^\circ - 37^\circ 22' 38,5'' = \boxed{142^\circ 37' 21,5''}$$

Área lateral "S" del arquimediano

Se compone de la suma de 20 caras triangulares y 12 caras pentagonales, ambas regulares y de lado "l"; la su-

The first part of the problem is to find the value of x which satisfies the equation $\log_2(x+1) = 3$.
 We can solve this by using the definition of logarithms. If $\log_a(b) = c$, then $a^c = b$.
 In this case, $a = 2$, $c = 3$, and $b = x+1$. So, $2^3 = x+1$.
 This simplifies to $8 = x+1$, which gives $x = 7$.

The second part of the problem is to find the value of y which satisfies the equation $\log_3(y-2) = 2$.
 Using the same definition of logarithms, if $\log_a(b) = c$, then $a^c = b$.
 Here, $a = 3$, $c = 2$, and $b = y-2$. So, $3^2 = y-2$.
 This simplifies to $9 = y-2$, which gives $y = 11$.

The third part of the problem is to find the value of z which satisfies the equation $\log_5(z+3) = 1$.
 Using the definition of logarithms, if $\log_a(b) = c$, then $a^c = b$.
 Here, $a = 5$, $c = 1$, and $b = z+3$. So, $5^1 = z+3$.
 This simplifies to $5 = z+3$, which gives $z = 2$.

perficie será:

$$\boxed{S} = 20 \times \frac{\sqrt{3}}{4} l^2 + 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} l^2 = \boxed{(5\sqrt{3} + 3\sqrt{25+10\sqrt{5}}) l^2} =$$

$$= (8.6602541 + 20,6457288) l^2 = 29,3059829... l^2$$

Volumen "V" del arquimediano

Se compone de la suma de 20 pirámides de base triangular y altura "C₃" y de 12 pirámides de base pentagonal regular y altura "C₅"; en valor será:

$$\boxed{V} = 20 \times \frac{\sqrt{3}}{4} l^2 \times \frac{C_3}{3} + 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} l^2 \times \frac{C_5}{3} = \frac{5\sqrt{3}}{3} l^2 \times \frac{3\sqrt{3} + \sqrt{15}}{6} l +$$

$$+ \frac{3\sqrt{25+10\sqrt{5}}}{3} l^2 \times \frac{\sqrt{5+2\sqrt{5}}}{5} l = \left(\frac{5 \times (3\sqrt{3}\sqrt{3} + \sqrt{15}\sqrt{3})}{18} + \right.$$

$$\left. + \frac{\sqrt{25+10\sqrt{5}} \times \sqrt{5+2\sqrt{5}}}{5} \right) l^3 = \left(\frac{5(9+3\sqrt{5})}{18} + \sqrt{\frac{(25+10\sqrt{5})(5+2\sqrt{5})}{5}} \right) l^3 =$$

$$= \left(\frac{5(3+\sqrt{5})}{6} + \sqrt{\frac{125+50\sqrt{5}+50\sqrt{5}+100}{5}} \right) l^3 = \left(\frac{5(3+\sqrt{5})}{6} + \sqrt{25+10\sqrt{5}+10\sqrt{5}+20} \right) l^3$$

$$= \left(\frac{5(3+\sqrt{5})}{6} + \sqrt{45+20\sqrt{5}} \right) l^3 = \left(\frac{5(3+\sqrt{5})}{6} + \sqrt{(9+4\sqrt{5}) \times 5} \right) l^3 =$$

$$= \left(\frac{5(3+\sqrt{5})}{6} + \sqrt{9+4\sqrt{5}} \times \sqrt{5} \right) l^3 = \left[\frac{5(3+\sqrt{5})}{6} + \left(\sqrt{\frac{10}{2}} + \sqrt{\frac{8}{2}} \right) \sqrt{5} \right] l^3 =$$

$$= \left[\frac{5(3+\sqrt{5})}{6} + (\sqrt{5}+2) \times \sqrt{5} \right] l^3 = \left(\frac{5(3+\sqrt{5})}{6} + 5+2\sqrt{5} \right) l^3 =$$

1. The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1801.

2. The second part is a report from the Secretary of the Treasury, dated January 1, 1801.

3. The third part is a report from the Secretary of the Navy, dated January 1, 1801.

4. The fourth part is a report from the Secretary of the War, dated January 1, 1801.

5. The fifth part is a report from the Secretary of the Interior, dated January 1, 1801.

$$= \frac{15 + 5\sqrt{5} + 30 + 12\sqrt{5}}{6} l^3 = \boxed{\frac{45 + 17\sqrt{5}}{6}} l^3 = 13,83\ 55\ 25\ 94 \dots l^3$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 20 triángulos equiláteros y 12 pentágonos regulares, de lado 34 mm, de forma que en cada vértice concurren 2 triángulos y 2 pentágonos.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\frac{\sqrt{5}+1}{2} l$	1.61 80 34.... l
b	$\frac{\sqrt{5}+2\sqrt{5}}{2} l$	1.53 88 42.... l
c_3	$\frac{3\sqrt{3} + \sqrt{15}}{6} l$	1.51 15 23.... l
c_5	$\frac{\sqrt{5}+2\sqrt{5}}{5} l$	1.37 63 82.... l
d_3	$\frac{\sqrt{3}}{3} l$	0.57 73 50.... l
d_5	$\frac{\sqrt{5}+\sqrt{5}}{10} l$	0.85 06 57.... l
m	$\frac{\sqrt{5}+\sqrt{5}}{8} l$	0.95 10 57.... l
α_3	$\lg \alpha_3 = 3 + \sqrt{5}$	$\lg \alpha_3 = 5,23\ 60\ 68$ $\alpha_3 = 79^\circ\ 11'\ 15,7''$
α_5	$\lg \alpha_5 = 2$	$\alpha_5 = 63^\circ\ 26'\ 5,8''$
φ_{3-5}	$\alpha_3 + \alpha_5$ $\lg \varphi_{3-5} = -(3 + \sqrt{5})$	$-\lg \varphi_{3-5} = 0,76\ 39\ 32$ $\varphi_{3-5} = 142^\circ\ 37'\ 21,5''$
S	$(5\sqrt{3} + 3\sqrt{25+10\sqrt{5}}) l^2$	29,30 59 83.... l^2
V	$\frac{45 + 17\sqrt{5}}{6} l^3$	13,83 55 26.... l^3

The following table shows the results of the experiment conducted on the 10th of May 1900. The results are given in the form of a table, and the data is as follows:

Time	Temperature	Pressure
10.00	10.0	10.0
10.10	10.1	10.1
10.20	10.2	10.2
10.30	10.3	10.3
10.40	10.4	10.4
10.50	10.5	10.5
11.00	10.6	10.6
11.10	10.7	10.7
11.20	10.8	10.8
11.30	10.9	10.9
11.40	11.0	11.0
11.50	11.1	11.1
12.00	11.2	11.2
12.10	11.3	11.3
12.20	11.4	11.4
12.30	11.5	11.5
12.40	11.6	11.6
12.50	11.7	11.7
13.00	11.8	11.8
13.10	11.9	11.9
13.20	12.0	12.0
13.30	12.1	12.1
13.40	12.2	12.2
13.50	12.3	12.3
14.00	12.4	12.4
14.10	12.5	12.5
14.20	12.6	12.6
14.30	12.7	12.7
14.40	12.8	12.8
14.50	12.9	12.9
15.00	13.0	13.0

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder, en la lámina 36, a la representación gráfica del arquimedeano IV.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotas complementarias cuyo cálculo efectuaremos posteriormente. Todas las magnitudes las obtendremos en función del lado " l_{IV} " del arquimedeano, cuya longitud es de 34 mm.

Calculemos previamente las siguientes magnitudes:

$$l_{IV} = (\text{dato del ejercicio}) = 34,0 \text{ mm}$$

$$a = 1,618034... \times 34 = 55,0 \text{ mm}$$

$$b = 1,538842... \times 34 = 52,3 \text{ mm}$$

$$C_3 = 1,511523... \times 34 = 51,4 \text{ mm}$$

$$C_5 = 1,376382... \times 34 = 46,8 \text{ mm}$$

$$d_3 = 0,577350... \times 34 = 19,6 \text{ mm}$$

$$d_5 = 0,850651... \times 34 = 28,9 \text{ mm}$$

El orden de operaciones del trazado gráfico (lámn. 36), es el siguiente:

1.º Situar el centro O , de coordenadas 72, 72, 85 mm

2.º Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio $a = 55 \text{ mm}$

3° Representar en I, II y III, la cara pentagonal, supuesto el poliedro colocado con dicha cara paralela a II, y un lado (3-1) perpendicular a I (utilizarse la cota " c_5 " en I y III, y la " d_5 " en II)

4° Obtener en I, las proyecciones del vértice 9 de la cara contigua triangular de arista 3-4 hasta colocar el vértice 9 sobre la esfera circunscrita. Para ello se hará centro en 3_I , con radio igual a la altura ^(h_p) de la cara 3-4-9 y se trazará un arco que corte en 9_I a la esfera circunscrita.

5° Determinar las proyecciones en II y III de dicho vértice 9, y seguidamente en I, II y III la de los vértices 6, 7, 8 y 10 (dichos vértices son a su vez de un pentágono regular de plano paralelo al II).

6° Determinar en II la posición de los vértices 11 al 20; esto son los de un decágono regular de lado " l_{10} " inscrito en una circunferencia de radio " a^4 "; los vértices 11 y 16 están sobre el eje paralelo al " Y ".

7° Obtener las proyecciones de los anteriores vértices 11 al 20, sobre I y III; el plano que los contiene es paralelo a II.

8° Obtener las proyecciones en II de los restantes vértices 21 al 31, sabiendo que son simétricos con respecto a un eje paralelo a Y, que pasa por O. Los n° 31, 22, 23, 24,

The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts. The second part of the paper is devoted to a discussion of the specific properties of the atom, such as its size, its mass, and its energy. It is shown that the specific properties of the atom are determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts.

The third part of the paper is devoted to a discussion of the specific properties of the atom, such as its size, its mass, and its energy. It is shown that the specific properties of the atom are determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts. The fourth part of the paper is devoted to a discussion of the specific properties of the atom, such as its size, its mass, and its energy. It is shown that the specific properties of the atom are determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts.

The fifth part of the paper is devoted to a discussion of the specific properties of the atom, such as its size, its mass, and its energy. It is shown that the specific properties of the atom are determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts. The sixth part of the paper is devoted to a discussion of the specific properties of the atom, such as its size, its mass, and its energy. It is shown that the specific properties of the atom are determined by the laws of quantum mechanics, and that the laws of quantum mechanics are in agreement with the experimental facts.

25, 26, 27, 28, 29, 30 se corresponderán respectivamente con los 10, 9, 8, 7, 6, 5, 4, 3, 2, 1. Con esta proyección, y conociendo la altura "f", se puede obtener fácilmente las proyecciones en I y III de los mencionados vértices 21 al 30. Las cotas "b", "c₃", "f" y "g" sirven de comprobación al trazado gráfico.

Como comprobación y necesaria ayuda para el trazado gráfico dado anteriormente, vamos a determinar analíticamente las siguientes magnitudes complementarias que darán mayor exactitud a dichos trazados (ver lám. 36).

Altura "n" de una cara triangular

Se demuestra en geometría, es

$$n = \frac{\sqrt{3}}{2} l = 0.8660254... l$$

Para el caso del dibujo, será: $n = 0.8660254 \times 34 = 29.4 \text{ mm.}$

Distancia "g" de los vértices 6 al 10 al plano de la cara pentagonal 1 al 5, y de los vértices 21 al 25 a la cara pentagonal 26 al 30

Se obtiene proyectando la altura "n" sobre el plano III;

1. The first part of the question is to find the value of x in the given figure. The figure shows a triangle with interior angles 40° , 60° , and x° . The sum of the interior angles of a triangle is 180° . Therefore, we have:

$$40^\circ + 60^\circ + x^\circ = 180^\circ$$

$$100^\circ + x^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 100^\circ$$

$$x^\circ = 80^\circ$$

2. The second part of the question is to find the value of y in the given figure. The figure shows a triangle with interior angles 50° , 70° , and y° . The sum of the interior angles of a triangle is 180° . Therefore, we have:

$$50^\circ + 70^\circ + y^\circ = 180^\circ$$

$$120^\circ + y^\circ = 180^\circ$$

$$y^\circ = 180^\circ - 120^\circ$$

$$y^\circ = 60^\circ$$

3. The third part of the question is to find the value of z in the given figure. The figure shows a triangle with interior angles 30° , 90° , and z° . The sum of the interior angles of a triangle is 180° . Therefore, we have:

$$30^\circ + 90^\circ + z^\circ = 180^\circ$$

$$120^\circ + z^\circ = 180^\circ$$

$$z^\circ = 180^\circ - 120^\circ$$

$$z^\circ = 60^\circ$$

4. The fourth part of the question is to find the value of w in the given figure. The figure shows a triangle with interior angles 20° , 110° , and w° . The sum of the interior angles of a triangle is 180° . Therefore, we have:

$$20^\circ + 110^\circ + w^\circ = 180^\circ$$

$$130^\circ + w^\circ = 180^\circ$$

$$w^\circ = 180^\circ - 130^\circ$$

$$w^\circ = 50^\circ$$

5. The fifth part of the question is to find the value of v in the given figure. The figure shows a triangle with interior angles 10° , 120° , and v° . The sum of the interior angles of a triangle is 180° . Therefore, we have:

$$10^\circ + 120^\circ + v^\circ = 180^\circ$$

$$130^\circ + v^\circ = 180^\circ$$

$$v^\circ = 180^\circ - 130^\circ$$

$$v^\circ = 50^\circ$$

6. The sixth part of the question is to find the value of u in the given figure. The figure shows a triangle with interior angles 15° , 135° , and u° . The sum of the interior angles of a triangle is 180° . Therefore, we have:

$$15^\circ + 135^\circ + u^\circ = 180^\circ$$

$$150^\circ + u^\circ = 180^\circ$$

$$u^\circ = 180^\circ - 150^\circ$$

$$u^\circ = 30^\circ$$

el ángulo de proyección es de

$$\varphi_{3.5} - 90^\circ = 142^\circ 37' 21.5'' - 90^\circ = 52^\circ 37' 21.5''$$

$$g = n \times \cos 52^\circ 37' 21.5'' = \frac{\sqrt{3}}{2} \times \cos 52^\circ 37' 21.5'' \times l =$$

$$= 0.52 \ 57 \ 31 \ 10 \dots \times l$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \frac{1}{2} l_3 3 &= \frac{1}{2} \times 0.477 \ 12 \ 13 \dots = 0.238 \ 56 \ 07 \\ + l_2 \cos 52^\circ 37' 21.5'' &= \dots = 7.783 \ 23 \ 30 \\ &\quad 0.021 \ 79 \ 37 \\ - l_2 2 &= \dots = 0.301 \ 03 \ 00 \\ l_2 0.52 \ 57 \ 31 \ 10 \dots &= 1.720 \ 76 \ 37 \end{aligned}$$

Para el caso del dibujo, será: $g = 0.52 \ 57 \ 31 \ 10 \dots \times 34 = 17.9 \text{ mm}$

Distancia "f" entre los dos planos paralelos a II, que contienen los vértices 6 al 10 y 21 al 25 respectivamente.

Se obtiene por diferencia de las alturas "c₅" y "g", ya calculadas.

$$\begin{aligned} f &= 2 (c_5 - g) = 2 \times (1.37 \ 63 \ 81 \ 9 - 0.52 \ 57 \ 31 \ 1) l = \\ &= 1.70 \ 13 \ 01 \ 6 \dots l \end{aligned}$$

Para el caso del dibujo, será: $f = 1.70 \ 13 \ 01 \ 6 \times 34 = 57.8$

Blank body area with faint horizontal lines.

Radio "r" de la circunferencia circunscrita al pentágono regular de vértices 6 al 10 y 21 al 25.

Es el cateto de un triángulo rectángulo de hipotenusa "a" y cateto " $\frac{f}{2}$ ". Su valor será:

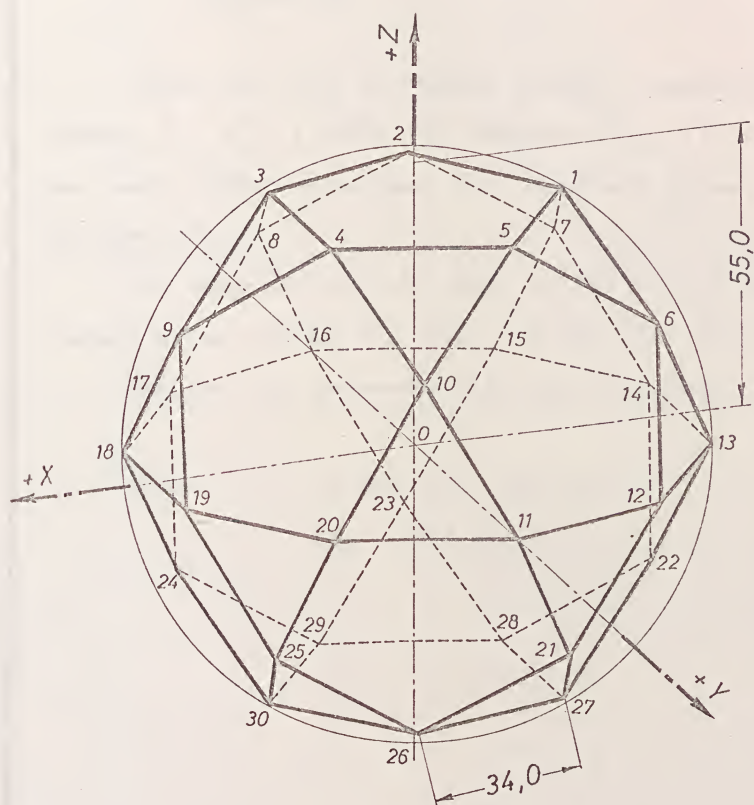
$$\boxed{r} = \sqrt{a^2 - \left(\frac{f}{2}\right)^2} = \sqrt{(4,61\ 20\ 34\ 0)^2 - (0,85\ 06\ 50\ 8)^2} = \boxed{1,37\ 63\ 81\ 9... l}$$

Para el caso del dibujo, $r = 46,8\text{ mm}$.

A continuación resumimos los valores anteriores.

CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
n	$\frac{\sqrt{3}}{2} l$	0,86 60 25.... l
g	$\frac{\sqrt{3}}{2} \cos 52^\circ 37' 21,5''$	0,52 57 31.... l
f	$2(C_5 - g)$	1,70 13 02.... l
r	$\sqrt{a^2 - \left(\frac{f}{2}\right)^2}$	1,37 63 82... l



Arquimedeano IV



Fig. 1. Dodecahedron.

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano V en el que en cada vértice concurren tres cuadrados y un triángulo equilátero

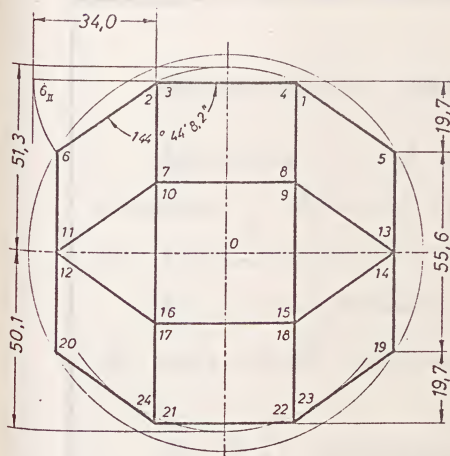
La longitud de su lado es 39.3 mm y las coordenadas de su centro O, son O (72, 72, 25) mm.

Dibujar en formato A3V y a escala 1:1.

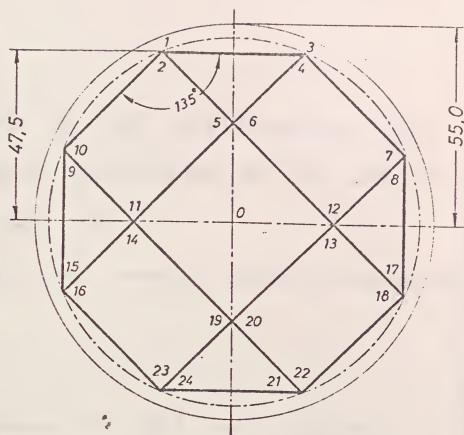
DATOS

O (72, 72, 25) mm

 $l_V = 39,3 \text{ mm}$

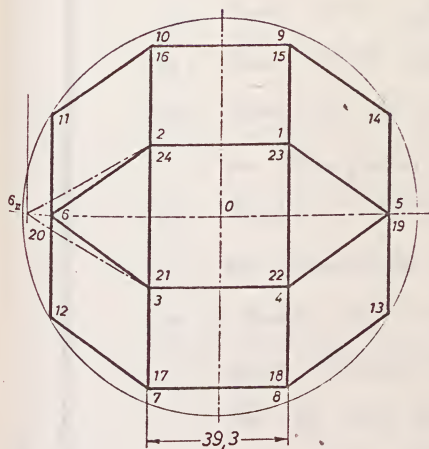
$$Z_+$$


III



+X

0

 $+Y.$ 

ARQUIMEDIANO V

Número de caras triangulares.....	$C_3 = 8$
Número de caras cuadradas.....	$C_4 = 18$
Número de vértices.....	$V = 24$
Número de aristas.....	$A = 48$
Número de caras de un ángulo sólido:	$1 C_3 + 3 C_4$

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquemediano V, en el que en cada vértice concurren un triángulo equilátero y tres cuadrados.

La longitud de su lado es de 39,3 milímetros y las coordenadas de su centro O son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

II

	Propuesta	De entrega	Entregada	Califi- cación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Arquimediano V					Lámina 37
1:1						Curso 10 10



The first diagram is a circle with an inscribed cube. The cube's edges are represented by lines connecting points on the circle's circumference. The lines are drawn in a perspective view, showing the front and side faces of the cube.

The second diagram is a circle with a complex internal grid structure. The grid consists of several horizontal and vertical lines, creating a series of rectangular cells within the circle. The lines are drawn in a perspective view, showing the front and side faces of the grid.



CONSIDERACIONES PREVIAS

Requiere en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el estudio del "Arquimediano I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

l = Arista del arquimedianos V (dato del ejercicio).

a = Radio de la esfera circunscrita.

b = Radio de la esfera tangente a las aristas.

c_3 = Radio de la esfera tangente a las caras triangulares.

c_4 = Radio de la esfera tangente a las caras cuadradas.

d_3 = Radio de la circunferencia circunscrita a una cara triangular.

d_4 = Radio de la circunferencia circunscrita a una cara cuadrada.

m = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

α_3 = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimedianos, que pasa por una arista de aquella.

α_4 = Ángulo rectilíneo del diedro formado por una cara cuadrada, con el plano diametral del ar-

The first of these is the fact that the
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...

The second of these is the fact that the
... of the ... of the ... of the ...
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... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...

The third of these is the fact that the
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...
... of the ... of the ... of the ...

quimedeano que pasa por una arista de aquélla.

φ_{3-4} = Ángulo rectilíneo del diedro formado por una cara triangular y otra cuadrada.

φ_{4-4} = Ángulo rectilíneo del diedro formado por dos caras cuadradas.

S = Superficie

V = Volumen

PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedeano, nos indica que se compone de 8 caras triangulares regulares y de 18 caras cuadradas; 24 vértices y 48 aristas.

En cada vértice concurren 3 cuadrados y un triángulo equilátero, todos de lados " l " iguales; por consiguiente concurrirán también 4 aristas del arquimedeano.

Así pues, tendremos que

$$\text{ARQUIMEDIANO V } (1 P_3 + 3 P_4); C_3 = 8; C_4 = 18; V = 24; A = 48$$

Cálculo de sus magnitudes

Arista " l " del arquimedeano

Dato del ejercicio

...

...

...

...

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las cuatro aristas de un ángulo sólido.

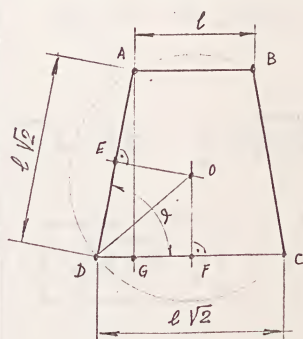


Figura 1

Este polígono es un trapecio isósceles A.B.C.D (fig. 1), cuya base menor AB es el lado "l" del arquimediano (lado de la cara triangular) y los otros tres lados AD, DC y CD, todos iguales, son las diagonales de las tres caras cuadradas que completan el

ángulo sólido de dicho arquimediano.

Si trazamos por E y F, puntos medios respectivos de los lados AD y DC, perpendiculares a éstos, dichas perpendiculares se cortarán en un punto O, centro de la circunferencia circunscrita al trapecio A.B.C.D, y de radio $OD = m$. Trazando seguidamente por A, la perpendicular a DC, se nos formará el triángulo rectángulo ADG, recto en G; en éste se verificará que

$$DG = \frac{DC - AB}{2} = \frac{l\sqrt{2} - l}{2} \quad \text{y también que}$$

$$\cos \alpha = \frac{DG}{AD} = \frac{l\sqrt{2} - l}{2} : l\sqrt{2} = \frac{l\sqrt{2} - l}{2\sqrt{2}l} = \frac{\sqrt{2} - 1}{2\sqrt{2}} = \frac{2 - \sqrt{2}}{4}$$

y sabiendo, por trigonometría, que

The following information is to be used in the solution of the problem. The information is given in the form of a table.

Table 1: Data for the problem. The table shows the values of the variables x and y for different values of t . The values of x and y are given in the first two columns, and the values of t are given in the third column.



The graph shows that the relationship between x and y is non-linear. The curve starts at the origin and increases as x increases. The curve is concave down, indicating that the rate of increase of y with respect to x is decreasing.

The following table shows the values of the variables x and y for different values of t .

x	y	t
0	0	0
1	1	1
2	4	2
3	9	3
4	16	4
5	25	5

$$\cos \frac{\alpha}{2} = \sqrt{\frac{1}{2} (1 + \cos \alpha)} \quad \text{se verificará}$$

$$\boxed{\cos \frac{\alpha}{2}} = \sqrt{\frac{1}{2} (1 + \cos \alpha)} = \sqrt{\frac{1}{2} \left(1 + \frac{2 - \sqrt{2}}{4}\right)} = \sqrt{\frac{1}{2} \times \frac{4 + 2 - \sqrt{2}}{4}} = \boxed{\frac{1}{2} \sqrt{\frac{6 - \sqrt{2}}{2}}}$$

Pero en la figura 1, vemos que $OD = \frac{FD}{\cos \frac{\alpha}{2}}$ y finalmente

$$\boxed{m} = \frac{l \sqrt{2}}{2}, \quad \frac{1}{2} \sqrt{\frac{6 - \sqrt{2}}{2}} = \frac{\sqrt{2}}{\sqrt{\frac{6 - \sqrt{2}}{2}}} l = \sqrt{\frac{2}{6 - \sqrt{2}}} l = \sqrt{\frac{4}{6 - \sqrt{2}}} l =$$

$$= \boxed{2 \sqrt{\frac{6 + \sqrt{2}}{34}}} l = 0,93394883... l$$

(Véase al final de este estudio, otro proceso para la determinación del radio "m")

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33), a este caso particular

$$\boxed{a} = \frac{l^2}{2 \sqrt{l^2 - m^2}} = \frac{l^2}{2 \sqrt{l^2 - \left(2 \sqrt{\frac{6 + \sqrt{2}}{34}} l\right)^2}} = \frac{l}{2 \sqrt{1 - 4 \times \frac{6 + \sqrt{2}}{34}}} =$$

$$= \frac{1}{2 \sqrt{1 - \frac{12 + 2\sqrt{2}}{17}}} l = \frac{1}{2 \sqrt{\frac{17 - 12 - 2\sqrt{2}}{17}}} l = \frac{1}{2 \sqrt{\frac{5 - 2\sqrt{2}}{17}}} l = \frac{1}{2} \sqrt{\frac{17}{5 - 2\sqrt{2}}} l =$$

Handwritten text in Devanagari script, likely a letter or document. The text is faint and mostly illegible due to blurring. It appears to be a formal communication, possibly a letterhead or a document header, with a date and a subject line. The text is written in a cursive style, typical of handwritten documents from the early 20th century.

$$= \frac{1}{2} \sqrt{\frac{17(5+2\sqrt{2})}{17}} l = \boxed{\frac{1}{2} \sqrt{5+2\sqrt{2}} l} = 1.39896633... l$$

(dibujo $a = 55 \text{ mm}$) $l = \underline{39.315 \text{ mm}}$

Radio "b" de la esfera tangente a las aristas.

Aplicando la fórmula general [3] (ver lám. 33), tendremos:

$$\boxed{b} = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{1}{2} \sqrt{5+2\sqrt{2}} \times l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{5+2\sqrt{2}}{4} - \frac{1}{4}} \cdot l =$$

$$= \sqrt{\frac{4+2\sqrt{2}}{4}} l = \boxed{\sqrt{\frac{2+\sqrt{2}}{2}} l} = 1.30656296... l$$

(dibujo: $b = 51.3 \text{ mm}$)

Radio "d₃" de la circunferencia circunscrita a una cara triangular de lado "l"

Se demuestra en geometría, es

$$\boxed{d_3 = \frac{\sqrt{3}}{3} l} = 0.57735027... l$$

(dibujo: $d_3 = 22.7 \text{ mm}$)

Radio "d₄" de la circunferencia circunscrita a una cara cuadrada.

Se demuestra en geometría, es

$$\boxed{d_4 = \frac{\sqrt{2}}{2} l} = 0.70710678... l$$

(en dibujo $d_4 = 27.8 \text{ mm}$)

Let x and y be two numbers such that $x + y = 10$ and $x - y = 2$. Find the values of x and y .

Solution: We are given two equations:

$$\begin{aligned} x + y &= 10 \quad \text{--- (1)} \\ x - y &= 2 \quad \text{--- (2)} \end{aligned}$$

Adding equation (1) and equation (2), we get:

$$(x + y) + (x - y) = 10 + 2$$

$$2x = 12$$

$$x = \frac{12}{2}$$

$$x = 6$$

Substituting the value of x in equation (1), we get:

$$6 + y = 10$$

$$y = 10 - 6$$

$$y = 4$$

Therefore, the values of x and y are $x = 6$ and $y = 4$.

Verification: Substituting $x = 6$ and $y = 4$ in equation (2), we get:

$$6 - 4 = 2$$

$$2 = 2$$

The LHS equals the RHS, so the solution is correct.

Thus, the solution to the system of equations is $x = 6$ and $y = 4$.

Final Answer: $x = 6$ and $y = 4$.

Radio " c_3 " de la esfera tangente a las caras triangulares regulares de lado " l "

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} C_3 &= \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\frac{1}{2} \sqrt{5+2\sqrt{2}} l\right)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{5+2\sqrt{2}}{4} - \frac{1}{3}} \cdot l = \\ &= \sqrt{\frac{15+6\sqrt{2}-4}{12}} \cdot l = \sqrt{\frac{11+6\sqrt{2}}{12}} \cdot l = \frac{\sqrt{11+6\sqrt{2}}}{2\sqrt{3}} l = \frac{\sqrt{\frac{18}{2}} + \sqrt{\frac{4}{2}}}{2\sqrt{3}} l = \\ &= \frac{3+\sqrt{2}}{2\sqrt{3}} l = \frac{3\sqrt{3} + \sqrt{6}}{6} l = 1,27427369 \cdot l \\ &\quad \text{(en dibujo: } c_3 = 50,1 \text{ mm)} \end{aligned}$$

Radio " c_4 " de la esfera tangente a las caras cuadradas de lado " l "

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} C_4 &= \sqrt{a^2 - (d_4)^2} = \sqrt{\left(\frac{1}{2} \sqrt{5+2\sqrt{2}} l\right)^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} = \sqrt{\frac{5+2\sqrt{2}}{4} - \frac{1}{2}} \cdot l = \\ &= \sqrt{\frac{5+2\sqrt{2}-2}{4}} \cdot l = \sqrt{\frac{3+2\sqrt{2}}{4}} l = \frac{\sqrt{3+2\sqrt{2}}}{2} l = \frac{\sqrt{\frac{4}{2}} + \sqrt{\frac{2}{2}}}{2} l = \frac{\sqrt{2}+1}{2} l = \\ &= 1,20710678... l \\ &\quad \text{(en dibujo: } c_4 = 47,5 \text{ mm)} \end{aligned}$$

Ángulo rectilíneo " α_3 " del diedro formado por una cara triangular, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Date	Page No.	Page No.
10/10/2020	10	10
11/10/2020	11	11
12/10/2020	12	12
13/10/2020	13	13
14/10/2020	14	14
15/10/2020	15	15
16/10/2020	16	16
17/10/2020	17	17
18/10/2020	18	18
19/10/2020	19	19

Se determina, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{t_3 \alpha_3} = \frac{2C_3}{\sqrt{4(d_3)^2 - l^2}} = \frac{2 \times \frac{3\sqrt{3} + \sqrt{6}}{6} l}{\sqrt{4\left(\frac{\sqrt{3}}{3}l\right)^2 - l^2}} = \frac{3\sqrt{3} + \sqrt{6}}{3\sqrt{4 \times \frac{1}{3} - 1}} = \frac{3\sqrt{3} + \sqrt{6}}{3\sqrt{\frac{1}{3}}} = \frac{(3\sqrt{3} + \sqrt{6}) \times \sqrt{\frac{1}{3}}}{3 \times \frac{1}{3}} = \frac{3\sqrt{3 \times \frac{1}{3}} + \sqrt{6 \times \frac{1}{3}}}{1} = \boxed{3 + \sqrt{2}} = 4, 41 \ 42 \ 13 \ 56$$

$$\hookrightarrow t_3 \alpha_3 = 0,64 \ 48 \ 53 \ 3$$

$$\boxed{\alpha_3 = 77^\circ \ 14' \ 8,2''}$$

Ángulo rectilíneo " α_4 " del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se determina, en función de su tangente, por la fórmula general [6] (ver lám. 33).

$$\boxed{t_4 \alpha_4} = \frac{2C_4}{\sqrt{4(d_4)^2 - l^2}} = \frac{2 \times \frac{\sqrt{2} + 1}{2} l}{\sqrt{4\left(\frac{\sqrt{2}}{2}l\right)^2 - l^2}} = \frac{\sqrt{2} + 1}{\sqrt{4 \times \frac{1}{2} - 1}} = \boxed{\sqrt{2} + 1} = 2, 41 \ 42 \ 13 \ 56 \dots$$

$$\hookrightarrow t_4 \alpha_4 = 0,382 \ 77 \ 57$$

$$\boxed{\alpha_4 = 67^\circ \ 30' \ 00''}$$

Ángulo rectilíneo " α_{3-4} " del diedro formado por una cara triangular regular y una cuadrada

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Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{3-4}} = \alpha_3 + \alpha_4 = 77^\circ 14' 8,2'' + 67^\circ 30' 00'' \\ = \boxed{144^\circ 44' 8,2''}$$

Ángulo rectilíneo " φ_{4-4} " del diedro formado por dos caras cuadradas.

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{4-4}} = 2 \alpha_4 = 2 \times 67^\circ 30' = \boxed{135^\circ}$$

Obsérvese que este valor es el del ángulo que forman dos lados consecutivos de un octógono regular, y en efecto, la proyección del arquimedianos, en el plano III, es un octógono regular de lado " l ".

Área lateral " S " del arquimedianos

Se compone de la suma de 8 caras triangulares regulares y de 18 caras cuadradas, ambas de lado " l "; la superficie será pues:

$$\boxed{S} = 8 \times \frac{\sqrt{3}}{4} l^2 + 18 l^2 = (2\sqrt{3} + 18) l^2 = \boxed{2 (\sqrt{3} + 9) l^2} = \\ = 21,46410162 \dots l^2$$



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Volumen "V" del arquimedianos

Se compone de la suma de 8 pirámides de base triangular y altura " c_3 " y de 18 pirámides de base cuadrada y altura " c_4 "; su valor será:

$$V = 8 \times \frac{\sqrt{3}}{4} l^2 \times \frac{c_3}{3} + 18 \times l^2 \times \frac{c_4}{3} = \frac{2\sqrt{3}}{3} \times \frac{3\sqrt{3} + \sqrt{6}}{6} l^3 + 6 \times \frac{\sqrt{2} + 1}{2} l^3 =$$

$$= \left(\frac{18 + 2\sqrt{18}}{18} + 3(\sqrt{2} + 1) \right) l^3 = \left(\frac{3 + \sqrt{2}}{3} + (3\sqrt{2} + 3) \right) l^3 = \frac{3 + \sqrt{2} + 9\sqrt{2} + 9}{3} l^3 =$$

$$= \frac{12 + 10\sqrt{2}}{3} l^3 = 8,71404521... l^3$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 8 triángulos equiláteros de lado 39,3 mm. y de 18 cuadrados de igual lado, de forma que en cada vértice concurren 3 cuadrados y 1 triángulo.

En el cuadro sinóptico que damos a continuación resumiremos los resultados analíticos obtenidos anteriormente.

The first part of the paper is devoted to a discussion of the general principles of the theory of the α -particle. It is shown that the α -particle is a helium nucleus, and that its mass is approximately four times that of a proton. The energy of the α -particle is also discussed, and it is shown that it is of the order of a few million electron volts.

The second part of the paper is devoted to a discussion of the properties of the α -particle. It is shown that the α -particle is a very penetrating particle, and that it is able to pass through several centimeters of air. It is also shown that the α -particle is a very ionizing particle, and that it is able to produce a large number of ion pairs per centimeter of air.

The third part of the paper is devoted to a discussion of the uses of the α -particle. It is shown that the α -particle is used in a number of different ways, including the measurement of the thickness of materials, the measurement of the density of materials, and the measurement of the concentration of materials. It is also shown that the α -particle is used in a number of different types of detectors, including the Geiger-Muller counter, the scintillation counter, and the proportional counter.

The fourth part of the paper is devoted to a discussion of the future of the α -particle. It is shown that the α -particle is a very important particle, and that it is likely to be used in a number of different ways in the future. It is also shown that the α -particle is a very interesting particle, and that it is likely to be the subject of a great deal of research in the future.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
a	$\frac{1}{2} \sqrt{5+2\sqrt{2}} \ell$	1. 39 89 66... ℓ
b	$\sqrt{\frac{2+\sqrt{2}}{2}} \ell$	1. 30 65 63... ℓ
C_3	$\frac{3\sqrt{3} + \sqrt{6}}{6} \ell$	1. 27 42 74... ℓ
C_4	$\frac{1+\sqrt{2}}{2} \ell$	1. 20 71 07... ℓ
d_3	$\frac{\sqrt{3}}{3} \ell$	0. 57 73 50... ℓ
d_4	$\frac{\sqrt{2}}{2} \ell$	0. 70 71 07... ℓ
m	$2\sqrt{\frac{6+\sqrt{2}}{34}} \ell$	0. 93 39 49... ℓ
α_3	$\text{tg } \alpha_3 = 3 + \sqrt{2}$	$\text{tg } \alpha_3 = 4. 41 42 14...$ $\alpha_3 = 77^\circ 14' 8.2''$
α_4	$\text{tg } \alpha_4 = 1 + \sqrt{2}$	$\text{tg } \alpha_4 = 2. 41 42 14...$ $\alpha_4 = 67^\circ 30'$
φ_{3-4}	$\alpha_3 + \alpha_4$	$\varphi_{3-4} = 144^\circ 44' 24.2''$
φ_{4-4}	$2 \alpha_4$	$\varphi_{4-4} = 135^\circ$
S	$2(\sqrt{3} + 9) \ell^2$	31, 46 41 02... ℓ^2
V	$\frac{12 + 10\sqrt{2}}{3} \ell^3$	8, 71 40 15... ℓ^3

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 37, a la representación gráfica del Arquimedianos V.

Para su trazado nos saldremos de cotas calculadas por las fórmulas anteriores y de procesos gráficos.

Calculemos previamente las siguientes magnitudes:

$$l_v = \text{Dato del ejercicio} = 39,3 \text{ mm}$$

$$a = 1,39 \ 89 \ 66... \times 39,3 = 55,0 \text{ mm}$$

$$b = 1,30 \ 65 \ 63... \times 39,3 = 51,3 \text{ mm}$$

$$c_3 = 1,27 \ 42 \ 74... \times 39,3 = 50,1 \text{ mm}$$

$$c_4 = 1,20 \ 71 \ 07... \times 39,3 = 47,5 \text{ mm}$$

$$d_3 = 0,57 \ 73 \ 50... \times 39,3 = 22,7 \text{ mm}$$

$$d_4 = 0,70 \ 71 \ 07... \times 39,3 = 27,8 \text{ mm}$$

El orden de operaciones del trazado gráfico (lámin. 37), es el siguiente:

- 1º Situar el centro O, de coordenadas $O(72, 72, 85) \text{ mm}$.
- 2º Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio $a = 55,0 \text{ mm}$.
- 3º Representar en I, II y III las caras cuadradas superior 1 al 4, e inferior 21 al 24, supuesto el poliedro colocado con dichas caras paralelas a II, y un lado perpendicular a I (utilícese la cota " c_4 " en I y III).
- 4º Obtener en I las proyecciones del vértice 6 de la cara contigua triangular, de arista 2-3, hasta colocar el vértice 6 sobre la esfera circunscrita. Para ello se hará centro en 3_2 , con radio igual a la altura " n " de la cara 6-2-3 (dibújese previamente en II), se

The following is a list of the names of the
 persons who have been named in the
 report of the committee on the
 subject of the proposed
 amendment to the
 constitution of the
 State of New York.
 The names are given in the
 order in which they were
 named in the report.

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 report.

trazará un arco que corte en c_1 a la esfera circunscrita.

5° Repítase la operación anterior para los vértices 5, 19, 30; obténgase seguidamente las proyecciones de los cuatro vértices anteriores, sobre II y III.

6° La determinación de los restantes vértices del poliedro, en I, II y III, es inmediata y no necesita explicación.

Obsérvese que el contorno aparente de la proyección III, es un octógono regular de lado "l", y que las proyecciones del poliedro en I y II, son iguales, aun cuando no sea coincidente la numeración de vértices en ambas.

Debido a la propiedad enunciada anteriormente, de ser el contorno de la proyección III del arquimedianos, un octógono regular de lado "l", puede comprobarse el cálculo de "b", igual al radio de la circunferencia circunscrita, y el de " c_4 ", igual al de la inscrita.

En efecto, tendremos que el ángulo central de un lado del octógono regular, valdrá:

$$\alpha_0 = \frac{360}{8} = 45^\circ \quad \text{y por lo tanto}$$

$$b = \frac{l}{2} : \operatorname{sen} \frac{\alpha_0}{2} = \frac{l}{2 \sqrt{\frac{1 - \cos \alpha_0}{2}}} = \frac{1}{\sqrt{4 \times \frac{1 - \frac{\sqrt{2}}{2}}{2}}} l = \sqrt{\frac{1}{2 \times (1 - \frac{\sqrt{2}}{2})}} l =$$

$$= \sqrt{\frac{1}{2 \times \frac{2 - \sqrt{2}}{2}}} l = \sqrt{\frac{1}{2 - \sqrt{2}}} l = \boxed{\sqrt{\frac{2 + \sqrt{2}}{2}}} l$$

y por otra parte

$$\begin{aligned} \boxed{C_4} &= \frac{l}{2} ; \frac{1}{5} \frac{\alpha_0}{2} = \frac{l}{2 \times \frac{1 - \cos \alpha_0}{\sin \alpha_0}} = \frac{1}{2 \times \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}} l = \frac{\frac{\sqrt{2}}{2}}{2 - \sqrt{2}} l = \\ &= \frac{\sqrt{2} (2 + \sqrt{2})}{2 \times (4 - 2)} l = \frac{2\sqrt{2} + 2}{4} l = \boxed{\frac{1 + \sqrt{2}}{2}} l \end{aligned}$$

valores ambos coincidentes con los ya calculados.

1870

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El cálculo que se incluye a continuación, para la determinación del radio "m" del Arquimediaco V, fue el que seguimos primeramente tomando como punto de partida el vértice A.

Como puede verse es mucho más laborioso que el seguido al tomar el vértice D, en el que el radio OD es bisectriz del ángulo D, cosa que no ocurre ~~con~~ con el radio OA. que ~~no es bisectriz~~ del ángulo A. ~~Lo es.~~

El resultado final es coincidente en ambos procesos, como lógicamente debía suceder.

Dear Mother, I am so glad to hear from you.

I am well and hope you are the same.

I have been thinking of you very much lately.

I hope you are happy and healthy.

I am sure you are. I love you very much.

I am sure you are. I love you very much.

I am sure you are. I love you very much.

I am sure you are. I love you very much.

I am sure you are. I love you very much.

I am sure you are. I love you very much.

de la [3] y [2], se deduce:

$$\boxed{\operatorname{sen} \gamma} = \frac{DG}{DA} = \frac{\sqrt{2}-1}{2} \ell; \ell \sqrt{2} = \frac{(\sqrt{2}-1) \ell}{2 \sqrt{2} \ell} = \frac{\sqrt{2}-1}{2 \sqrt{2}} = \boxed{\frac{2-\sqrt{2}}{4}} \quad [4]$$

y siendo $\varphi = \gamma + \frac{\pi}{2}$ será $\cos \varphi = -\operatorname{sen} \gamma$, por lo que

$$\boxed{\cos \varphi} = -\operatorname{sen} \gamma = -\frac{2-\sqrt{2}}{4} = \boxed{\frac{\sqrt{2}-2}{4}} \quad \text{y por lo tanto} \quad [5]$$

$$\boxed{\operatorname{sen} \varphi} = \sqrt{1 - \cos^2 \varphi} = \sqrt{1 - \left(\frac{\sqrt{2}-2}{4}\right)^2} = \sqrt{1 - \frac{2+4-4\sqrt{2}}{16}} = \sqrt{1 - \frac{6-4\sqrt{2}}{16}} =$$

$$= \sqrt{1 - \frac{3-2\sqrt{2}}{8}} = \sqrt{\frac{8-3+2\sqrt{2}}{8}} = \boxed{\sqrt{\frac{5+2\sqrt{2}}{8}}} \quad [6]$$

De la figura se deduce:

$$AO = \boxed{m} = \frac{AF}{\cos \beta} = \boxed{\frac{\ell}{2 \cos \beta}} \quad [7]$$

y también

$$AO = \boxed{m} = \frac{AE}{\cos(\varphi-\beta)} = \boxed{\frac{\ell \sqrt{2}}{2 \cos(\varphi-\beta)}} \quad [8]$$

De la [7] y [8]

$$\frac{\ell}{2 \cos \beta} = \frac{\ell \sqrt{2}}{2 \cos(\varphi-\beta)} \quad \cos(\varphi-\beta) = \sqrt{2} \cos \beta$$

$$\cos \varphi \cos \beta - \operatorname{sen} \varphi \operatorname{sen} \beta = \sqrt{2} \cos \beta$$

$$\cos \varphi \cos \beta - \operatorname{sen} \varphi \sqrt{1 - \cos^2 \beta} = \sqrt{2} \cos \beta \quad [9]$$

si hacemos en [9] $\cos \beta = x$; $\cos \varphi = p$; $\operatorname{sen} \varphi = q$

tendremos:

CC

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Blank body area with faint horizontal lines.

$$p x - q \sqrt{1-x^2} = \sqrt{2} x \quad " \quad (p - \sqrt{2}) x = q \sqrt{1-x^2} ;$$

$$p - \sqrt{2} = q \sqrt{\frac{1-x^2}{x^2}} \quad " \quad \frac{p - \sqrt{2}}{q} = \sqrt{\frac{1}{x^2} - 1} \quad " \quad \left(\frac{p - \sqrt{2}}{q}\right)^2 = \frac{1}{x^2} - 1 ;$$

$$\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1 = \frac{1}{x^2} \quad " \quad x^2 = \frac{1}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1} \quad \text{de donde.}$$

$$x = \sqrt{\frac{1}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}} = \cos \beta \quad [10]$$

valor que sustituido en [7], nos da

$$m = \frac{l}{2 \sqrt{\frac{1}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}}} = \frac{1}{\sqrt{\frac{4}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}}} l = \sqrt{\frac{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}{4}} \times l =$$

$$= \frac{1}{2} \sqrt{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1} \times l \quad [11]$$

sustituyendo en ésta los valores $p = \cos \varphi$; $q = \sin \varphi$,
obtenidos en [5] y [6], tendremos :

$$\left(\frac{p - \sqrt{2}}{q}\right)^2 = \left(\frac{\sqrt{2} - 2}{4} - \sqrt{2}\right)^2 : \frac{5 + 2\sqrt{2}}{8} = \left(\frac{\sqrt{2} - 2 - 4\sqrt{2}}{4}\right)^2 : \frac{5 + 2\sqrt{2}}{8} =$$

$$= \frac{-(3\sqrt{2} + 2)^2}{16} : \frac{5 + 2\sqrt{2}}{8} = \frac{18 + 4 + 12\sqrt{2}}{16} : \frac{5 + 2\sqrt{2}}{8} =$$

$$= \frac{22 + 12\sqrt{2}}{16} : \frac{10 + 4\sqrt{2}}{16} = \frac{22 + 12\sqrt{2}}{10 + 4\sqrt{2}} = \frac{11 + 6\sqrt{2}}{5 + 2\sqrt{2}} = \frac{(11 + 6\sqrt{2})(5 - 2\sqrt{2})}{17} =$$

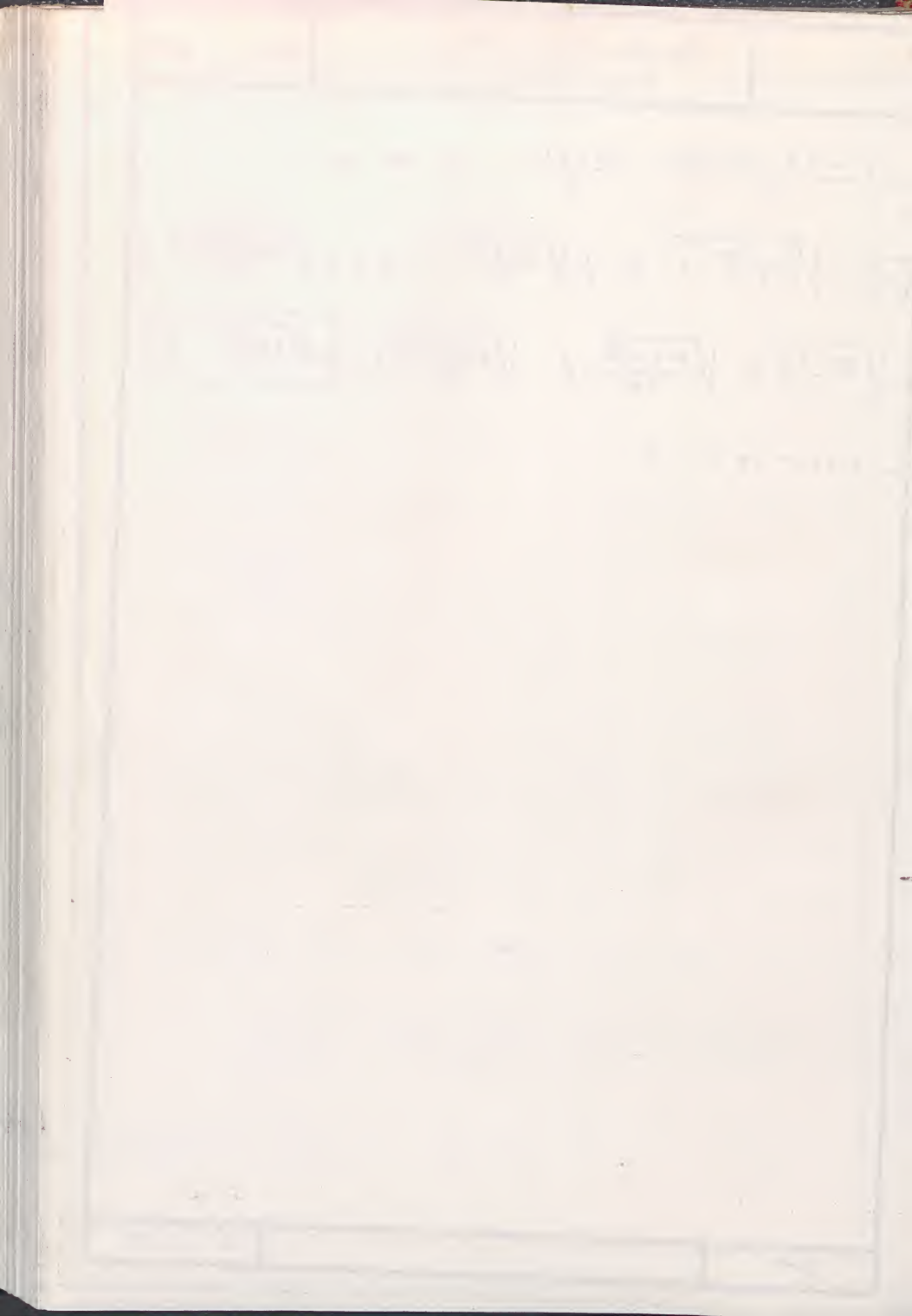
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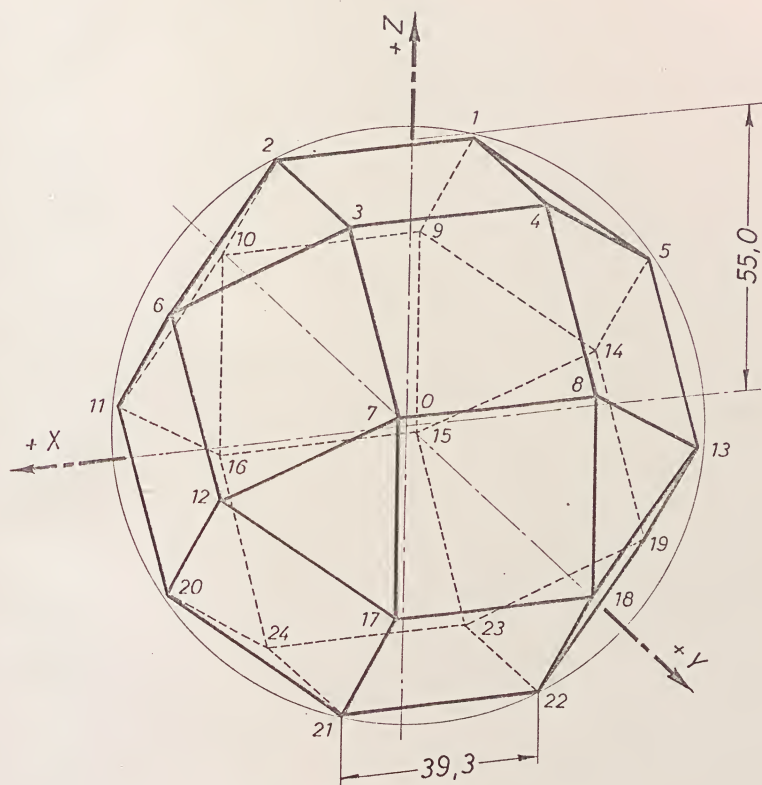
$$= \frac{55 + 30\sqrt{2} - 22\sqrt{2} - 24}{17} = \frac{31 + 8\sqrt{2}}{17} \quad \text{y de aquí:}$$

$$m = \frac{1}{2} \sqrt{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1} \cdot l = \frac{1}{2} \sqrt{\frac{31 + 8\sqrt{2}}{17} + 1} \cdot l = \frac{1}{2} \sqrt{\frac{31 + 8\sqrt{2} + 17}{17}} \cdot l =$$

$$= \sqrt{\frac{48 + 8\sqrt{2}}{4 \times 17}} \cdot l = \sqrt{\frac{34 + 4\sqrt{2}}{34}} \cdot l = \sqrt{\frac{4(6 + \sqrt{2})}{34}} \cdot l = \boxed{2 \sqrt{\frac{6 + \sqrt{2}}{34}} \cdot l}$$

$$= 0.93394883 \dots l$$





Arquimediano V



1. Introduction





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